The Economics of Data Spaces: Inter-Platform Externalities and Governance

Doriani Lingga and Damien Mayaux

Abstract

Data spaces are networks of independent and interoperable platforms with shared rules enabling their participants to exchange data. They exhibit inter-platform externality: participants value cross-side interactions both within and across platforms to a different extent. However, platforms price independently, leading to an inefficient level of participation. We develop a model of two-sided platforms to study how data space governance mechanisms address this coordination problem. We find that a monopolist outperforms several forms of decentralized governance. A price coordination mechanism can improve on this benchmark while making the platforms better off.
Keywords: data space, data-sharing governance, inter-platform externality, two-sided market
JEL: L13, L51, L86

1 Introduction

Data is a nonrival good (Jones and Tonetti, 2020): the same dataset can be used by many firms at once, without being used up. It is therefore efficient to share data widely. In a growing number of industries, this sharing is organized by data spaces. A data space is a network of two-sided platforms that intermediate between data providers and data consumers, on the basis of interoperability standards and a common set of governance rules. CATENA-X, for example, is the data space of the German automotive industry, and Siemens and BMW are both members of it. Siemens uses it to supply IoT sensor data to BMW and to Mercedes-Benz, while BMW uses it to obtain sensor data from Siemens and from Bosch.

Data spaces have become an instrument of industrial policy. China announced in 2024 an action plan to create one hundred data spaces in the country by 2028. The European Union launched its Data Union Strategy in 2025, and funds pilot data spaces in specific industrial sectors across the continent. Given the scale of these commitments, little is known about how data spaces should be designed.

The economics of data spaces lies less inside any one intermediary than between them. Data intermediaries emerge locally, where trust and sectoral proximity are strongest, but the interactions they enable extend beyond their boundaries. For instance, French car manufacturers may also benefit from the sensor data that Siemens supplies through CATENA-X. We call *inter-platform externality* the benefit that a participant derives from participation on the other side of a different platform. The ambition of the European project of a "Data Union" is precisely to leverage this positive externality at scale.

This inter-platform externality creates a coordination problem, hence a need for governance mechanisms. Network effects operate at the level of the data space, but prices are set by each platform independently. Without coordination, each platform therefore ignores the value that its own members create for the participants of other platforms, cross-platform participation is under-provided, which is socially inefficient and detrimental for the platforms themselves. Platform concentration would be one approach to address this coordination problem, but some industries may be reluctant to engage in data-sharing with tech giants. Data spaces offer an alternative route: they keep the platforms separate and independently owned, while minimizing the gap with rules rather than with ownership.

In this paper, we propose several governance mechanisms that a data space could adopt in the presence of inter-platform externalities. A governance mechanism is understood as a rule that defines how and by whom access fees are set in the data space. We model a data space as two interoperable platforms, one in each of two regions. Each platform serves participants on the two sides of the market: data providers and data consumers. Every potential participant can join the platform of their own region. Data consumers may pay an additional fee to also join the platform of the other region. Participants benefit from cross-side participation, but they value within-platform interactions more than cross-platform ones. Participants also have a stand-alone valuation of participation to data-sharing, uniformly distributed and independent of the number of platforms that they join. Participation is determined as a fulfilled-expectations equilibrium.

Our model has two features that depart from the usual literature on two-sided platforms. The first one is the absence of business stealing. Because every participant is attached to their home platform, the two platforms never compete for the same participant. This feature describes well how data spaces connect complementary communities, rather than fighting over a common pool of participants. The second feature is that the inter-platform externality is strictly lower than the intra-platform one. Studies on platform merger or interoperability tend to consider that platforms are perfect substitutes for each other, an assumption that cannot reasonably be carried over to the data-sharing context.

We compare different scenarios under which prices could be set. In the absence of coordination, each platform would set the fees to be paid by the participants joining their platform - namely, the base fee for data consumers and providers from their region, as well as the additional fee for the consumers from the other region. We call this governance mechanism *the destination principle*. We take it as the baseline, because it is often found in data spaces that have barely considered coordination on business issues. We examine two benchmarks, *the social planner* and *the monopoly* scenarios.

We then consider two alternative governance mechanisms, the *origin principle* and the *customs union* cases. The origin principle is similar to the destination principle, except that the crossing fee is set and collected by the home platform of the consumer paying it. Under the customs union, the platforms are offered the option of charging a single fee to home participants and no additional fee for joining the other platform. The customs union can be seen as a social planner constrained by the participation decision of the platforms. Given that these governance mechanisms are more demanding and that the data space has no direct control over the platforms, we introduce coordination as a preliminary stage in which either both platforms accept the mechanism and prices are set accordingly, or at least one refuses and prices are set according to the destination principle. All the five scenarios considered (social planner, monopolist, destination principle, origin principle and customs union) admit closed-form solutions in participation, prices and welfare.

First, in the five scenarios considered, all data consumers multi-home. The reason is that we assume that data consumers are heterogeneous in their taste for data-sharing, but homogeneous in their taste for joining the platform in the other region. Therefore, profit-maximizing firms prefer to set prices compatible with multi-homing, which induces a higher participation abroad and fuels back in the form of more value for their home participants. Under the destination principle, the additional fee for joining the other platform is such that data consumers are indifferent between single-homing and multi-homing. Under the other four scenarios, the distinction between the base price and the additional fee is irrelevant as only their sum - the total price paid by data consumers - matters.

Second, it matters which platform sets and collects the cross-platform fee. A data space adopting the destination principle is worse than under the origin principle, leading to higher prices, lower participation and welfare for both sides of the market. Since the origin principle leads to higher profit than the destination principle baseline, this governance mechanism is always accepted by the platforms when the data space proposes it. This theoretical result challenges the conventional rationale for the prevalence of the destination principle among data spaces.

Third, a monopoly outperforms both the destination principle and the origin principle. A single owner of both platforms charges strictly lower prices, induces higher participation on both sides and in both regions, earns a higher profit, and delivers strictly higher welfare to both sides of the market. The reason is that two independent platforms fail to internalize the positive effect of their prices on the rival platform's provider participation. A monopolist, by contrast, internalizes this spillover: because it is unambiguously positive, doing so increases the marginal value of participation and therefore induces lower prices. This highlights the limitations of data spaces relying on loose forms of business governance, such as the destination or origin principle.

Fourth, a data space with price coordination can yield higher welfare than a monopoly. Suppose that the data space proposes a single pair of prices to both platforms and participation clears the market. We show that the welfare-maximizing level of participation induced by optimal prices lies strictly beyond the profit-maximizing one, because the data space deliberately sacrifices platform rent, down to the platforms' outside option, in exchange for participation. The customs union falls short of the first best only because the participation constraint leaves the platforms a strictly positive rent. In summary, the customs union comes closest to the first best, followed by the monopoly and then the origin principle. The baseline destination principle is associated with the lowest welfare level.

We extend our main results by introducing asymmetries in the model, where we revisit the five scenarios and analyze more carefully their distributional implications. We focus on two types of asymmetries in terms of cross-side externalities: across regions and across sides. Across regions, we consider the case where cross-region interactions benefit the participants from each region unequally. Across sides, we consider the case where data providers and data consumers differ in their valuation of the interaction. We also analyze what changes when data providers incur a positive cost to join their home platform. Our extension reveals that a monopolist, just like the social planner, tends to minimize the effects of the asymmetry. It leaves participants in the two regions equally well off however unequally they value the link, absorbing the whole asymmetry into where it collects its rent, and it equalizes the two sides unless one of them bears a participation cost. Both the destination and the origin principle instead pass the asymmetries directly into unequal fees, hence, unequal participation and surplus. They are therefore not only less efficient, but also lack the ability to minimize the asymmetry and redistribute the rents. We show that the customs union can cancel cross-side asymmetries and can set prices that specifically maximize the welfare of data providers.

Our paper offers, to the best of our knowledge, the first mathematical model of the economics of data spaces. We use it to address fundamental questions on the business governance of data spaces that have practical implications for the current wave of development of these organizations across the globe, such as in China and in the European Union.

Related literature. The paper connects three strands of literature. The first is the theory of two-sided markets (Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006; Rysman, 2009; Hagiu, 2009), in which cross-side network effects run between the two sides of a given platform. Multi-homing (Teh et al., 2023; Bakos and Halaburda, 2020), within-group externalities (Belleflamme and Peitz, 2019; Lin et al., 2025) and negative cross-group externalities (Anderson, Foros, and Kind, 2018; Sandrini and Somogyi, 2023) have been studied extensively in various contexts in that domain. Existing models typically treat cross-side externalities as invariant to the platform affiliation of the counterpart. We relax this assumption by allowing the strength of the externality to differ between same-platform and cross-platform interactions and trace its consequences. One implication is that value flows across platform boundaries in our model even in the absence of multi-homing, once free interconnection is imposed.

The second strand is interoperability, interconnection and platform mergers. Interoperability is the technical precondition for a data space (Kerber and Schweitzer, 2017; Ekmekci, White, and Wu, 2025; Krämer and Shekhar, 2025; Kim, 2026; Laatikainen and Tuikka, 2025). Concern about concentration in digital markets puts interoperability mandates on the policy agenda (European Commission. Directorate General for Competition., 2019; Motta and Peitz, 2024). We take interoperability as given and explore the economic governance to be layered on top of it, since inter-platform externalities are a consequence of interoperability. The merger literature (Evans and Noel, 2008; Correia-da-Silva et al., 2019; Lin et al., 2025; Cornière and Taylor, 2024) weighs the internalization of network effects against the loss of competition. Our monopoly result isolates the first force by removing the second, and thereby identifies a class of settings, namely interconnected and non-competing platforms, in which consolidation is unambiguously beneficial. Closest in spirit is Carballa-Smichowski et al. (2026), who argue that platforms often act as complements rather than as competitors, and who identify the mechanisms that bind them into an "inter-platform ecosystem". Their work is primarily conceptual, whereas we supply a parameterized model and solve it. The broader literature on platform ecosystems (Jacobides, Cennamo, and Gawer, 2018; Jacobides, Cennamo, and Gawer, 2024; Parker, Alstyne, and Choudary, 2016) is likewise largely organizational.

The third strand is the economics of data and of data spaces. Work on the value and the nonrivalry of data (Jones and Tonetti, 2020; Veldkamp and Chung, 2024; Farboodi and Veldkamp, 2021; Bergemann, Bonatti, and Smolin, 2018) establishes why data sharing is efficient, while work on data-driven market power (Prüfer and Schottmüller, 2021; Motta and Peitz, 2021) explains why it may nonetheless fail to happen. Data spaces themselves have so far been studied mainly through institutional and conceptual lenses (Eustache, Brousseau, and Toledano, 2025; Martens, 2024; Richter, 2023; Hutterer, Krumay, and Muehlburger, 2023; Wysel, Baker, and Billingsley, 2021). To our knowledge, this paper is the first to formally model a data space as a governed pair of interconnected two-sided platforms, propose several governance rules, and rank the candidate rules by the welfare that they deliver.

The paper is organized as follows. Section 2 sets out the model, solves the participation stage, and then characterizes the price equilibrium of a data space under the destination principle, our baseline, together with the social planner and monopoly benchmarks. Section 3 turns to the other governance mechanisms a data space may propose and analyzes the origin principle and the customs union. Section 4 introduces asymmetries in network externalities across regions and sides and investigates their distributional consequences under the previously considered scenarios. Section 5 concludes.

2 The model

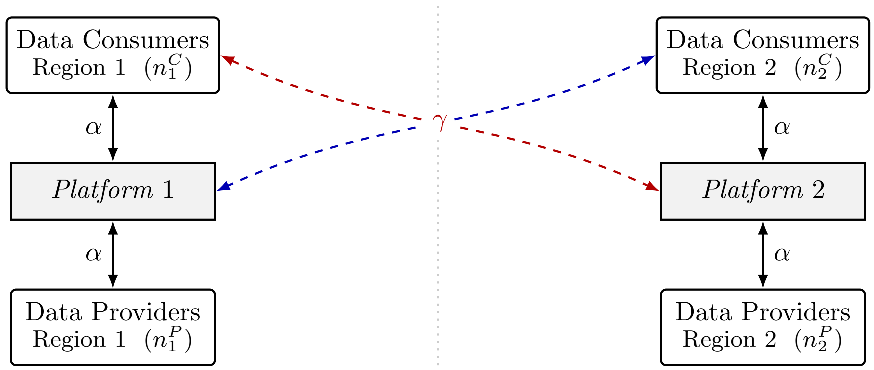
Two platforms are located in two regions, 1 and 2, each named after its region. Each platform is a data intermediary serving the two sides of its market: data providers (P) and data consumers (C). Interoperability is costless, and each platform can serve and set different fees for participants from within and from outside its region.

We allow data consumers to multi-home but restrict data providers to single-home. This asymmetric homing structure reflects the distinct roles of the two sides in data-sharing ecosystems and allows us to isolate the role of inter-platform externalities. Because consumers can multi-home, providers can reach consumers from the neighboring region without joining the neighboring platform themselves.

We also assume that data consumers can join the platform in the other region only after joining their local platform. This reflects the fact that interoperability is typically easier to achieve between intermediaries through common standards than directly between a consumer and a foreign intermediary. More broadly, proximity in geographical distance, language, and trust may discourage consumers from joining a foreign platform directly.

Figure 1 summarizes the forms of data-sharing within and across regions in our model, as well as their valuation by the participants.

Figure 1: Data-sharing within and across Platforms



Notes: Within a region, providers and consumers who participate on their own platform value the marginal interaction at α. Consumers may additionally incur a crossing fee τ to join the other region's platform, and value the marginal interaction there at $\gamma < \alpha$. Providers can only single-home, so cross-region value reaches them only through the consumers who cross, and they equally value cross-region interactions at γ.

Notations: Throughout, $k, l \in \{1, 2\}$ with $l \neq k$. The superscripts C and P refer to the two sides of the market, respectively, data consumers and data providers.

- n_k^P: mass of participating data providers from region k.
- n_k^C: mass of participating data consumers from region k.
- n_{kl}^C: mass of participating data consumers from region k who also join platform l.
- n_{kk}^C: mass of participating data consumers from region k who do not join platform l.
- p_k^P: fee paid by participating data providers from region k to platform k.
- p_k^C: base fee paid by participating data consumer from region k to platform k.
- τ_{kl}^C: crossing fee, the additional fee paid by participating data consumers from region k to also join platform l, on top of the base fee p_k^C. Section 3 varies which platform sets and collects τ_{kl}^C.

- $T_{kl}^C := p_k^C + \tau_{kl}^C$: total payment made by participating data consumers from region k who are also joining platform l.
- α: marginal value of a cross-side interaction with a participant from the same region.
- γ: marginal value of a cross-side interaction with a participant from the other region.

Every active participant benefits from cross-side interactions, and values an interaction with participants from their own region more than with those from the other region. In particular, they value cross-side, same-platform interactions at α, and cross-side, cross-platform interactions at $\gamma < \alpha$. Denote n_k^s as the mass of side $s \in \{C, P\}$ active participants from region $k \in \{1, 2\}$, and n_{lk}^C as the mass of region-l consumers who multi-home by crossing into platform k. A data provider in region k obtains the following utility:

$$U_k^P = v^P + \alpha n_k^C + \gamma n_{lk}^C - p_k^P,$$

and a data consumer in region k obtains

$$U_{kk}^C = v^C + \alpha n_k^P - p_k^C \qquad \text{by joining platform } k \text{ only, and} \qquad U_{kl}^C = v^C + \alpha n_k^P + \gamma n_l^P - T_{kl}^C$$

by joining both platforms, where $T_{kl}^C = p_k^C + \tau_{kl}^C$. The stand-alone utility of participating is independent and uniformly distributed on each side, $v^P, v^C \sim U[0, 1]$. The mass of potential participants on each side of each region is normalized to one, and they join whenever their utility is nonnegative. Membership fees are denoted by p_k^s, corresponding to the fee paid by side s participants from region k for participating on platform k, and τ_{kl}^C, corresponding to the crossing-fee paid by a multi-homing consumer from region k. Note the structural asymmetry between the two sides: by crossing, a consumer gains access to all the providers in both regions, whereas a provider gains cross-region access only through foreign consumers who choose to multi-home.[1]

The profit of platform k is its revenue from the fees it sets and collects. Later in the paper, we study different data space governance mechanisms, which differ in who sets and collects the fees. Under every mechanism, the platform collects $p_k^P n_k^P$ from its own providers and $p_k^C n_k^C$ from its own consumers. The crossing revenue accrues to whichever platform the mechanism assigns it to.

The timing of the game is as follows. In the first stage, the platforms simultaneously choose the fees they control. In the second stage, data consumers and data providers decide whether or not to join the platform of their region, and conditional on joining, data consumers decide whether or not to also join the neighboring platform. The introduction of a data space, formalized and analyzed in Section 3, adds a stage zero in which an orchestrator proposes a governance mechanism that the platforms are free to accept or reject. We solve the game using backward induction, and our solution concept is sub-game perfect Nash equilibrium.

We introduce the following assumption to the analysis.

Assumption 1 (Weaker cross-border externality and interiority)**.** $0 < \gamma < \alpha$ *and* $\alpha \leq \frac{1}{5}$.

The first inequality is substantive: cross-region interactions are worth strictly less than same-region interactions, which is what distinguishes our environment from the standard multi-homing model. The second bounds how strong network effects are, and it constrains α alone. This keeps the

[1]Most platform models, including the existing work on platform mergers, effectively assume $\gamma = \alpha$. Our main contribution is to examine implications of the externality across platform boundaries, that is, the case $0 < \gamma < \alpha$.

participation system stable and ensures that equilibrium participation is interior (i.e. $n_k^P < 1$ and $n_k^C < 1$) in all the scenarios we study, except the first best. We show later that $\frac{1}{5}$ is essentially the weakest round bound on α that works under the "customs union" scenario of Section 3.2, the one for which we find the highest participation after the first best.

The rest of this section solves the model in the absence of any proposed governance mechanism. Section 2.1 defines the participation equilibria, and Section 2.2 expresses the relevant equations to be used across regimes. Section 2.3 solves the baseline model, denoted as the destination principle, in which the platforms are interoperable but each unilaterally prices access to its own participant base. Sections 2.4 and 2.5 present the two benchmarks against which every governance mechanism proposed will be measured: the social optimum, and a single owner of both platforms. Section 3 then introduces the data space, which proposes rules at stage zero which the platforms may accept or refuse, with the baseline payoff as their outside option. A summary of the different scenarios considered is provided in Table 1.

2.1 Participation equilibria

A participation equilibrium is a belief about the participation of others such that the individual decision of data providers whether or not to join their platform, as well as the decision of data consumers to single-home, multi-home or not joining, leads to an actual participation consistent with this belief.

Definition 1 (Participation equilibrium)**.** Given a price vector $p = \left(p_1^P, p_1^C, \tau_{12}^C, p_2^P, p_2^C, \tau_{21}^C\right)$, a participation equilibrium is a participation vector $n(p) = \left(n_1^P(p), n_{11}^C(p), n_{12}^C(p), n_2^P(p), n_{22}^C(p), n_{21}^C(p)\right)$ such that, for all $(k,l) \in \{(1,2),(2,1)\}$,

$$\begin{cases} n_k^P = \mathbb{P}(\ U_k^P = \max(U_k^P, 0)\ ) \\ n_{kk}^C = \mathbb{P}(\ U_{kk}^C = \max(U_{kk}^C, U_{kl}^C, 0)\ ) \\ n_{kl}^C = \mathbb{P}(\ U_{kl}^C = \max(U_{kk}^C, U_{kl}^C, 0)\ ) \end{cases}$$

In the definition, note that the probabilities are taken over the stand-alone values of individual providers (in the first case) and consumers (in the two other cases).

Now that the notion of equilibrium has been introduced, we investigate a consumer's decision whether or not to cross. A consumer from region k crosses whenever $U_{kl}^C \geq U_{kk}^C$, that is whenever

$$\gamma\, n_l^P \ \geq\ \tau_{kl}^C. \tag{1}$$

Condition (1) does not involve v^C, implying that the decision of a consumer whether or not to cross depends on prices and participation alone, whereas the decision whether or not to participate also involves the stand-alone utility of participating. All consumers of a region therefore make the same crossing decision: either all cross, or none does. We adopt the convention that they cross when indifferent.

In all the five price-setting scenarios studied in this paper (see Table 1), the relevant case is one where all consumers cross, as explained in the following remark.

Remark 1 (Price-setting leads to multi-homing)**.** *If the agent which sets the cross-border fee τ_{kl}^C has an objective function increasing in all participation levels (n_k^C, n_l^C, n_k^P and n_l^P), it will set a fee at which all data consumers multi-home. The reason is that inducing crossing costs the fee-setter nothing: lowering the crossing fee to the threshold γn_l^P makes consumers cross while leaving them exactly indifferent, and the cross-region interactions thereby created raise every participation level, and with them any objective increasing in participation.*

Remark 1 says which configuration prevails, but not at what fee. When the crossing fee is chosen to maximize a profit, its level is pinned down as well, as stated in the remark below.

Remark 2 (Profit-maximizing prices make consumers indifferent between single-homing and multi-homing)**.** *If the agent which sets the cross-border fee τ^C_{kl} has a profit function under multi-homing that is increasing in the cross-border fee, it will set it at a level that makes data consumers indifferent between crossing and not crossing.*

The above remarks are made precise, scenario by scenario, within the proofs of Appendix B. Together they imply that in every scenario we study all consumers multi-home, and it becomes our default configuration throughout the paper. We can, therefore, define the equilibrium under a given governance mechanism.

2.2 Prices, participation and surplus

When all active consumers multi-home, participation on each side is determined by a marginal participant: a potential participant joins whenever their stand-alone value exceeds the fee they pay net of the network benefits they enjoy. With stand-alone values uniform on $[0, 1]$, the mass of participants on platform k is given by:

$$n^P_k = 1 - p^P_k + \alpha\, n^C_k + \gamma\, n^C_l, \tag{2}$$
$$n^C_k = 1 - p^C_k - \tau^C_{kl} + \alpha\, n^P_k + \gamma\, n^P_l. \tag{3}$$

Participation is thus affine in fees, and the coefficient matrix is invertible under Assumption 1 (Appendix A gives the matrix and its determinant). The participation system can equivalently be written in inverse-demand form:

$$p^P_k = 1 - n^P_k + \alpha\, n^C_k + \gamma\, n^C_l, \qquad T^C_{kl} = 1 - n^C_k + \alpha\, n^P_k + \gamma\, n^P_l. \tag{4}$$

Under regional symmetry, writing n^P, n^C without indices for the common levels, we have:

$$p^P = 1 - n^P + (\alpha + \gamma)\, n^C, \qquad T = 1 - n^C + (\alpha + \gamma)\, n^P. \tag{5}$$

The above expressions imply that a side is priced according to the value it enjoys rather than the value it creates. In particular, the fee a side pays rises with the benefit $(\alpha + \gamma)$ per participant of the other side that its members draw from the platform. Any optimization problem in any relevant scenario can therefore be solved equivalently either over fees or over participation.

In a symmetric equilibrium, the crossing revenue a platform receives equals the crossing revenue its own consumers pay. Therefore, platform k's profit at a symmetric price equilibrium is $\Pi = p^P n^P + T n^C$ regardless of which agent collects the crossing fee. Substituting (5) into the platform's profit, we have:

$$\Pi(n^P, n^C) = n^P + n^C - (n^P)^2 - (n^C)^2 + 2(\alpha + \gamma)\, n^P n^C. \tag{6}$$

Total welfare is the sum of consumer surplus, provider surplus and platform profit, noting that prices are pure transfers.

Proposition 1 (Welfare and surplus under symmetry across regions)**.** *When participating consumers from both regions multi-home and participation is interior on both sides, provider and consumer surplus as well as total welfare are*

$$W^P = \big(n^P\big)^2, \qquad W^C = \big(n^C\big)^2, \qquad W = 2\,\Pi + \big(n^P\big)^2 + \big(n^C\big)^2, \tag{7}$$

and hence

$$W(n^P, n^C) = 2\Big[n^P + n^C - \tfrac{(n^P)^2}{2} - \tfrac{(n^C)^2}{2} + 2(\alpha+\gamma)\, n^P n^C\Big]. \tag{8}$$

When divided by region, the surplus of each group is half of the above: region k's providers obtain $(n_k^P)^2/2$ *and its consumers* $(n_k^C)^2/2$*, so that the regional welfare* W_k *is given by*

$$W_k = \tfrac{1}{2}\big(n_k^P\big)^2 + \tfrac{1}{2}\big(n_k^C\big)^2 + \Pi_k. \tag{9}$$

When participation is symmetric across the two sides of the market (i.e. $n^P = n^C = n$*), then* $\Pi(n,n) = 2n - 2\Delta n^2$ *and* $W(n,n) = 4n - 2\big(1 - 2(\alpha+\gamma)\big)n^2$*, where* $\Delta := 1 - \alpha - \gamma$*. Under Assumption 1,* $W(n,n)$ *is strictly increasing in* n *on* $[0,1]$*, while* $\Pi(n,n)$ *increases on* $\big[0, \frac{1}{2\Delta}\big]$ *and decreases beyond.*

Proof. See Appendix B. □

Proposition 1 has a simple logic. A participant with stand-alone utility v gains $v - \tilde{v}$ relative to the marginal participant $\tilde{v}$, who is exactly indifferent. With uniform stand-alone utility value, summing these gains over participants yields the linear-demand surplus triangle of area $n^2/2$ per side and per region. Fees and network effects therefore affect a group's surplus only by changing its own participation level. Ranking mechanisms by welfare hence amounts to ranking them by participation, and the regional decomposition stated in (9) extends the same logic to the distributional questions of Section 4.

We conclude this subsection by another argument applicable to most of the scenarios considered in this paper.

Remark 3 (Split irrelevance)**.** *If the same agent collects both* p_k^C *and* τ_{kl}^C *(which are by definition imposed on consumers from region k), the agent's profit and total welfare depend on them only through* $T_{kl}^C = p_k^C + \tau_{kl}^C$*. Any split with* $0 \le \tau_{kl}^C \le \gamma n_l^P$ *leads to the same outcomes (participation, surplus, ...), including when* $\tau_{kl}^C = 0$*.*

It is therefore convenient to treat the total cost of multi-homing T_{kl}^C as a single relevant instrument, rather than splitting it into p_k^C and τ_{kl}^C, when both are set by the same agent; carrying the split would only add a redundant degree of freedom. Indeed, the only scenario in which the split does matter is our baseline model, the destination principle, where the two fees are set by different platforms.

2.3 Price equilibrium with the destination principle

In the baseline, the crossing fee is set and collected by the platform the consumer joins in addition to their own: platform k sets τ_{lk}^C for region-l consumers entering platform k. We call this scenario the destination principle, where the levy accrues to the jurisdiction in which the good is consumed. We use it as the baseline model because it is the most likely outcome when interoperability is technically feasible but no business governance rule has been agreed within the data space, so each platform unilaterally prices access to its own participant base.

Under this scenario, platform k has three instruments $\big(p_k^P, p_k^C, \tau_{lk}^C\big)$ and maximizes

$$\Pi_k = p_k^P n_k^P + p_k^C n_k^C + \tau_{lk}^C\, n_l^C$$

taking the rival's fees as given. By Remark 1, platform k benefits from inducing region-l consumers to cross, and by Remark 2, it charges them up to the value they obtain from crossing, so $\tau_{lk}^C = \gamma n_k^P$ prevails in equilibrium. The crossing fee is therefore a constrained instrument, and the location of the constraint moves with platform k's own choices: lowering p_k^P raises n_k^P, which relaxes the cap and allows a higher charge on foreign consumers. The shadow value of the binding cap consequently enters the first-order conditions on the base fees.

Since $T_{kl}^C = p_k^C + \tau_{kl}^C$ and the caps bind in both regions, the equilibrium fees satisfy

$$p_k^C = 1 - n_k^C + \alpha\, n_k^P, \qquad \tau_{kl}^C = \gamma\, n_l^P, \qquad T_{kl}^C = 1 - n_k^C + \alpha\, n_k^P + \gamma\, n_l^P. \tag{10}$$

The base fee is priced off the own-region externality α alone, as the cross-region value γn_l^P that a home consumer obtains is extracted in full by the rival's crossing fee and is unavailable to the home platform. The total payment of a consumer thus carries two margins set by two independent platforms, representing a form of double marginalization which, as we show below, falls on the consumer side only.

Proposition 2 (Price equilibrium with the destination principle)**.** *The unique region-symmetric price equilibrium under the destination principle corresponds to the participation*

$$n^{C,\text{Dest}} = \frac{1-\alpha^2+\alpha\big(1-\alpha^2+\gamma^2\big)}{2\big(1-\alpha^2\big)-\gamma^2-2\alpha(\alpha+\gamma)\big(1-\alpha^2+\gamma^2\big)}, \qquad n^{P,\text{Dest}} = \tfrac{1}{2} + (\alpha+\gamma)\, n^{C,\text{Dest}}.$$

and the fees

$$p^{P,\text{Dest}} = \tfrac{1}{2}, \qquad p^{C,\text{Dest}} = 1 - n^{C,\text{Dest}} + \alpha\, n^{P,\text{Dest}}, \qquad \tau^{\text{Dest}} = \gamma\, n^{P,\text{Dest}}, \qquad T^{\text{Dest}} = p^{C,\text{Dest}} + \tau^{\text{Dest}}.$$

Moreover,

$$n^{P,\text{Dest}} - n^{C,\text{Dest}} = \tfrac{1}{2} - (1-\alpha-\gamma)\, n^{C,\text{Dest}} > 0 \text{ and } \tau^{\text{Dest}} - p^{C,\text{Dest}} = -\big(1-n^{C,\text{Dest}}\big) - (\alpha-\gamma)\, n^{P,\text{Dest}} < 0.$$

Proof. See Appendix B. □

The provider fee of one half is the fee that an isolated monopolist facing a one-sided linear demand would set, which holds independently of α and γ, and is in fact part of every best response, whatever the rival charges. The reason is that the platform levying the crossing fee captures the entire value created by one additional home provider: α per home consumer through the base fee and γ per foreign crosser through the crossing fee, which the binding cap allows it to raise by exactly γ per additional provider. Hence, on the provider side, each platform prices exactly as a monopolist owning both platforms would.

Distortion occurs on the consumer side. The home platform cannot price the cross-region value its consumers enjoy, since it has been extracted by the rival's crossing fee, so consumer participation is held below provider participation, $n^{P,\text{Dest}} > n^{C,\text{Dest}}$. The destination principle is the only scenario studied in this paper in which the symmetric equilibrium is associated with different participation levels on both sides, with consumers participating less than providers.

The second inequality of the proposition states that foreign consumers are nonetheless charged less per head than domestic ones, $\tau^{\text{Dest}} < p^{C,\text{Dest}}$. Two forces act on the crossing fee: the weaker cross-region externality pushes it down, while its role as a pure extraction margin pushes it up. The first force turns out to dominate throughout the admissible parameter range.

2.4 Socially optimal participation

In this section, we analyze the prices a social planner would assign to maximize total welfare. Since participation generates positive value for both consumers and providers and entails no social costs, as prices only redistribute surplus, it is socially optimal for all participants to join. The social planner's program is therefore straightforward.

Proposition 3 (First best participation)**.** *The socially optimal allocation is full participation and universal crossing, $n_k^P = n_k^C = 1$, implemented by zero prices, $p_k^P = p_k^C = \tau_{kl}^C = 0$, leading to total welfare*

$$W^{\text{Best}} = 2 + 4(\alpha + \gamma).$$

Proof. Evaluating (8) at $n^P = 1$ and $n^C = 1$ gives $W = 2[2 - 1 + 2(\alpha + \gamma)] = 2 + 4(\alpha + \gamma)$. □

Note that the first best is a corner, so the inversion (5) does not apply to it: at $n = 1$, the marginal participant is strictly better off participating, and any $p \leq \alpha + \gamma$ implements full participation, zero prices included. It implies that the first best does not pin down fees, rather, any common fee up to $\alpha + \gamma$ sustains it; moving fees within that range is a pure transfer from participants to the platforms, leaving W^{Best} unchanged. Table 2 reports the zero-fee implementation, which is the natural benchmark. Even at the most generous admissible fee, $\alpha + \gamma$, platforms earn less than their outside option Π^{Dest}, making the first best unattainable under any voluntary mechanism.

2.5 Monopoly

Another benchmark is the monopoly case, in which a single entity owns both platforms and optimizes prices across the two regions to maximize its total profit. Just like the social planner, the monopolist internalizes the inter-platform externality when setting its prices.

Proposition 4 (Monopoly outcome)**.** *A monopolist owning both platforms sets*

$$p^{P,\text{Mono}} = T^{\text{Mono}} = \tfrac{1}{2}$$

and achieves

$$n^{P,\text{Mono}} = n^{C,\text{Mono}} = \frac{1}{2\Delta}, \qquad \Pi^{\text{Mono}} = \frac{1}{2\Delta}, \qquad W^{\text{Mono}} = \frac{2\Delta + 1}{2\Delta^2} \qquad \textit{where} \qquad \Delta = 1 - \alpha - \gamma$$

Proof. See Appendix B. □

Note that the possible splits include the case of free interconnection, $\tau = 0$. In Proposition 4, Π^{Mono} denotes, as everywhere in the paper, the profit collected per platform, so that the monopolist's joint profit is $2\Pi^{\text{Mono}} = 1/\Delta$. Any split $T^{\text{Mono}} = p^{C,\text{Mono}} + \tau^{\text{Mono}}$ with $\tau^{\text{Mono}} \in [0, 1/2]$ and $p^{C,\text{Mono}} = 1/2 - \tau^{\text{Mono}}$ is equivalent.

The monopoly price is exactly one half on both sides, independently of α and γ. The externality parameters do not disappear, but rather, are absorbed entirely into the participation level $1/2\Delta$, which is increasing in both.

Whenever $\gamma > 0$, relative to the destination principle, monopoly delivers a lower total consumer price, higher participation on both sides, higher joint profit and higher welfare. The comparison is part of Proposition 6 below. The mechanism is the usual internalization argument, sharpened by the absence of business stealing. Each independent platform ignores that a lower price raises the rival's provider participation through γ, whereas the monopolist internalizes this spillover. Because no participant ever switches platforms, there is no competitive harm to weigh against the internalization gain. Unlike the ambiguous comparison prevalent in the platform-merger literature, here, consolidation dominates on every dimension. Any governance mechanism that keeps the platforms independently owned must therefore be measured against this benchmark.

Table 1: Summary of the five scenarios

Scenario name	Tag	Type	Who sets p_k^C and p_k^P?	Who sets τ_{kl}^C?
Destination principle	Dest	Fully decentralized	Platform k	Platform l
Origin principle	Orig	Partly decentralized	Platform k	Platform k
Customs union	Cust	Partly decentralized	Data space	Data space*
Social planner	Best	Centralized	Data space*	Data space*
Monopoly	Mono	Centralized	Monopolist	Monopolist

Notes: The first three scenarios are data space governance mechanisms and the last two serve as benchmark cases. The "Fully decentralized" case corresponds to the absence of active governance in the data space and serves as the baseline model. In the "Partly decentralized" cases, the two platforms are free to adopt the proposed governance mechanism or to revert to the "Fully decentralized" baseline. The star (*) indicates cases where the data space sets the corresponding price to zero. The "Tag" column gives the superscript used throughout the text to identify quantities associated with each scenario (e.g. Π^{Dest}, $n^{P,\text{Mono}}$, ...).

3 Data space governance mechanisms

In the previous section, the coordination problem remains unresolved. Left to themselves, technically interoperable platforms follow the destination principle, and each ignores the value its participants create on the other platform. A single owner would internalize this externality, but data spaces are designed precisely to avoid such concentration by allowing independently owned platforms to operate under common rules. This section therefore asks how far such rules alone can close the gap between decentralized outcomes and the monopoly and planner benchmarks when platforms remain free to walk away.

We now add stage zero to the game, where a data space orchestrator proposes a rule and both platforms either accept or refuse it. By accepting, they agree to be constrained by the fee rule set by the orchestrator, and by refusing, they revert to the destination-principle baseline of Section 2.3. The profit Π^{Dest} of Proposition 2 is therefore the outside option in all the other mechanisms, and no rule leaving either platform with a lower profit can be implemented. For the origin principle, Proposition 6 verifies that this participation constraint is satisfied strictly, whereas the customs union of Section 3.2 is constructed to satisfy it with equality. Table 1 summarizes how the mechanisms differ from the baseline and from the benchmarks.

3.1 Price equilibrium with origin principle

Under the origin principle, the crossing fee is set and collected by the consumer's home platform, so platform k controls $\left(p_k^P, p_k^C, \tau_{kl}^C\right)$. This arrangement departs from the standard two-sided market logic, in which participants typically pay the platform hosting the cross-side participants they wish to reach. In a data space context, this is a natural option since the crossing consumers would normally have already transacted with their home platform, which can bill access to the partner platform on their behalf and settle with it under the interoperability agreement.[2] By Remark 3 the platform's payoff depends on its two consumer-side instruments only through their total T_{kl}^C, so it effectively chooses $\left(p_k^P, T_{kl}^C\right)$, taking the rival's fees as given.

[2]This mechanism is also prevalent in telecommunications, where the network that originates a call bills its own subscriber.

The platform can equivalently choose its own participation pair $\left(n_k^P, n_k^C\right)$, holding fixed the rival's *fees* as opposed to the rival's participation: when platform k moves, the rival's participation responds through the network externality. Differentiating (2)–(3) for region l at fixed $\left(p_l^P, T_{lk}^C\right)$,

$$\begin{pmatrix} \mathrm{d}n_l^P \\ \mathrm{d}n_l^C \end{pmatrix} = \rho \begin{pmatrix} \alpha & 1 \\ 1 & \alpha \end{pmatrix} \begin{pmatrix} \mathrm{d}n_k^P \\ \mathrm{d}n_k^C \end{pmatrix}, \qquad \rho := \frac{\gamma}{1-\alpha^2}. \tag{11}$$

The regions decouple exactly when $\gamma = 0$, in which case $\rho = 0$ and each platform is an isolated two-sided monopolist. Economically, ρ measures how much one platform's success spills over to its partner: when platform k attracts more participants, joining becomes more valuable for the participants from region l, in proportion to the cross-region externality γ; the rival's own two-sided feedback then amplifies this impulse by factor of $1/(1-\alpha^2)$. An independent platform does not price this spillover, while the governance mechanisms below are designed to recover it.

Proposition 5 (Price equilibrium with the origin principle)**.** *Under Assumption 1, the symmetric equilibrium under the origin principle is*

$$n^{P,\mathrm{Orig}} = n^{C,\mathrm{Orig}} =: n^{\mathrm{Orig}} = \frac{1}{2-2\alpha-\gamma-\dfrac{\gamma^2}{1-\alpha}} \quad \text{and} \quad p^{P,\mathrm{Orig}} = T^{\mathrm{Orig}} = \frac{1-\alpha+\gamma}{2-2\alpha+\gamma}.$$

Proof. See Appendix B. ☐

The denominator of the first expression can be read term by term, taking 2 as the reference point, which is the value it takes when the two platforms are isolated one-sided monopolies. Each subtracted term lowers the denominator, hence raises participation and lowers the fee: 2α reflects the platform's internalization of its own two-sided feedback; γ the cross-region gain that it captures through its own crossing consumers; and $\gamma^2/(1-\alpha)$ the feedback through the rival's participation response shown in (11). An origin platform still does not internalize the gain accruing to the rival's providers, and this is exactly its gap to the monopolist.

Proposition 6 (Origin versus destination principles)**.** *Whenever $\gamma > 0$,*

$$p^{P,\mathrm{Dest}} = p^{P,\mathrm{Mono}} = \tfrac{1}{2} < p^{P,\mathrm{Orig}}, \qquad T^{\mathrm{Mono}} < T^{\mathrm{Orig}} < T^{\mathrm{Dest}},$$

$$n^{C,\mathrm{Dest}} < n^{\mathrm{Orig}} < n^{P,\mathrm{Dest}} < n^{\mathrm{Mono}}, \qquad \Pi^{\mathrm{Dest}} < \Pi^{\mathrm{Orig}} < \Pi^{\mathrm{Mono}}, \qquad W^{\mathrm{Dest}} < W^{\mathrm{Orig}} < W^{\mathrm{Mono}}.$$

All inequalities collapse to equalities at $\gamma = 0$, where the two regions decouple.

Proof. See Appendix B. ☐

The profit and welfare rankings admit a simple reading. Under the destination principle, the total consumer payment carries two margins set by two independent platforms, whereas under the origin principle, a single platform sets both, so one layer of double marginalization disappears, leading to a lower total consumer price and higher profits and welfare. The asymmetry between the two sides also disappears: the origin principle restores $n^P = n^C$ since the remaining margin no longer falls on consumers alone.

The participation ordering, however, is not uniform across sides. The destination principle prices providers exactly as the monopolist does ($p^P = \frac{1}{2}$), because the crossing fee allows the platform to capture the value its providers create for foreign crossers. The origin platform earns nothing from the foreign consumers who cross into it, so it has a weaker incentive to attract providers and hence charges them more. Since $W^P = (n^P)^2$ by Proposition 1, providers are strictly better off under the destination principle. Therefore, moving from the destination to the origin principle benefits the platforms and the consumers, but hurts the providers. We return to this point in Section 4, where the provider side emerges as the side that policymakers may have the strongest reason to protect.

Since both platforms are strictly better off under the origin principle than under their outside option, a data space proposing it at stage zero meets no resistance. The proposition also settles what a bare interconnection mandate achieves.

Proposition 7 (Free interconnection reproduces the origin principle)**.** *Mandating $\tau^C_{12} = \tau^C_{21} = 0$ without constraining base prices yields exactly the equilibrium of Proposition 5, at no cost to the platforms.*

Proof. By Remark 3, $\tau^C_{kl} = 0$ is one of the origin-principle platform's own optimal choices, since its payoff depends on the consumer-side instruments only through T^C_{kl} and any split with $0 \leq \tau^C_{kl} \leq \gamma n^P_l$ implements the same outcome. Removing the instrument therefore removes nothing the platform was using; and at $\tau^C_{kl} = 0$ the crossing condition (1) holds for any $n^P_l \geq 0$, so all consumers multi-home. □

Forbidding a separate charge for crossing thus buys the improvement of Proposition 6 and nothing beyond it: the outcome remains strictly short of monopoly, and further short of the first best.

3.2 Customs union

The origin principle only reassigns who sets an existing fee; a more ambitious data space could coordinate the level of the fees themselves, taking pricing — and not merely standards — into its remit.

We reserve the term customs union for the mechanism in which the data space imposes $\tau^C_{12} = \tau^C_{21} = 0$ and a common base price pair (p^P, p^C), subject only to each platform preferring to accept rather than revert to the baseline. Since the inversion (5) maps price pairs to participation pairs bijectively, the data space can equivalently be described as choosing (n^P, n^C) directly, subject to the platforms' participation constraint

$$\Pi(n^P, n^C) \;\geq\; \Pi^{\text{Dest}}. \tag{12}$$

Since (6) is a strictly concave quadratic, the feasible set is an ellipse together with its interior, centred on the unconstrained maximizer of Π — the monopoly point of Proposition 4. The data space thus maximizes W, which is increasing in both arguments by Proposition 1, over this ellipse. The optimum lies on the boundary, and because both Π and W are invariant under exchanging n^P and n^C, the unique optimum is fixed by that exchange: it is the point where the boundary meets the diagonal $n^P = n^C$.

Proposition 8 (Welfare-maximizing customs union)**.** *The welfare-maximizing price pair consistent with both platforms accepting is symmetric across sides and leads to*

$$n^{P,\text{Cust}} = n^{C,\text{Cust}} = \frac{1+\sqrt{S}}{2\Delta}, \qquad p^{P,\text{Cust}} = p^{C,\text{Cust}} = \frac{1-\sqrt{S}}{2}$$

where $\Delta := 1-\alpha-\gamma$ and $S := 1 - 2\Delta\,\Pi^{\text{Dest}} \in (0,1)$. Participation is interior, $n^{\text{Cust}} \leq 1$, if and only if $\Pi^{\text{Dest}} \geq 2(\alpha+\gamma)$, which Assumption 1 guarantees; and since $S > 0$, $n^{\text{Cust}} > n^{\text{Mono}}$.

Proof. See Appendix B. □

The customs union improves on monopoly for a simple reason: the monopolist maximizes profit and stops at the top of the profit function, whereas the data space is willing to trade platform rent for participation, and does so until the platforms are driven down to their outside option. The equilibrium falls short of the first best because the outside option is out of reach at full participation: any fee vector sustaining $n = 1$ yields each platform at most $2(\alpha + \gamma)$, strictly below Π^{Dest} under Assumption 1, so neither platform would accept. The customs union can thus be read as a social planner constrained by the platforms' voluntary participation.

3.3 The welfare ranking

Proposition 9 (Welfare ranking)**.** *Whenever $\gamma > 0$,*

$$W^{\text{Dest}} \ < \ W^{\text{Orig}} \ < \ W^{\text{Mono}} \ < \ W^{\text{Cust}} \ < \ W^{\text{Best}}.$$

Proof. By Proposition 1, W is strictly increasing along the diagonal on $[0, 1]$, so for the four mechanisms whose outcome lies on the diagonal - meaning that the two sides participate equally - the ranking follows from $n^{\text{Orig}} < n^{\text{Mono}} < n^{\text{Cust}} < 1$, established in Propositions 6 and 8. The destination outcome is off-diagonal; the comparison $W^{\text{Dest}} < W^{\text{Orig}}$ is part of Proposition 6, proved in Appendix B. □

Figure 2 summarizes graphically the five scenarios in the participation space. The feasible set of the customs union is the ellipse $\Pi \geq \Pi^{\text{Dest}}$; the destination outcome lies on its boundary by construction, while the origin and monopoly outcomes lie strictly inside, since both yield the platforms a profit above the baseline.

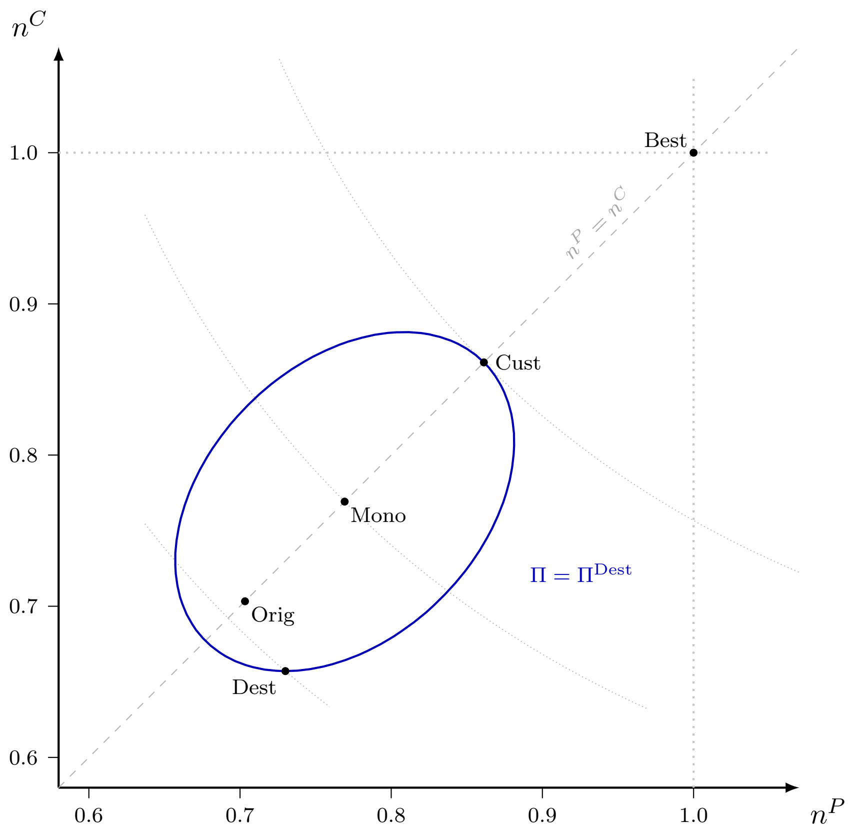
Three features of the figure are worth noting. First, the customs union does not expropriate the platforms: Dest and Cust lie on the same iso-profit curve, because the data space proposes fees so that the platforms are exactly as well off as under the destination-principle baseline, which is why they accept the mechanism at stage zero. Second, the customs union nonetheless improves on monopoly in terms of welfare, since the data space converts platform rent into participation while the monopolist retains it. Third, the first best is unreachable, even by a customs union: at $(1, 1)$ each platform's profit is at most $\Pi(1, 1) = 2(\alpha + \gamma)$, which falls strictly short of the outside option Π^{Dest} under Assumption 1 — this is the interiority condition of Proposition 8, which holds with slack — so $(1, 1)$ lies strictly outside the feasible ellipse $\Pi \geq \Pi^{\text{Dest}}$. Table 2 reports the numerical values associated with the five scenarios for the same values of the parameters as Figure 2.

Figure 3 reports numerical welfare values for the whole admissible range of γ, with the same value of α as in Figure 2. Welfare is increasing in γ across scenarios and the ordering of scenarios reflects Proposition 9. Note that the gap between the origin, destination, monopoly and customs union shrinks as γ tends to zero. In the limit case where $\gamma = 0$, the gap between these four scenarios and the first best reflects the deadweight loss under two regional monopolists. In the limit case where $\gamma = \alpha$, a gap between the customs union and the first best remains: full participation leaves each platform at most $2(\alpha + \gamma)$, strictly less than the outside option Π^{Dest}, so the acceptance constraint keeps the customs union short of the corner.

4 Asymmetry and distributional consequences

So far the two regions have been identical and the two sides have valued a match equally, so the only distributional question was between platforms and participants. In data spaces, however, one

Figure 2: The five scenarios in the participation space



Notes: Numerical results for $\alpha = 0.20$ and $\gamma = 0.15$. The ellipse is the platforms' participation constraint $\Pi = \Pi^{\text{Dest}}$, the feasible set of the customs union; the dotted arcs are iso-welfare curves. Mono is its centre, the unconstrained profit maximum. Dest lies on the ellipse by construction and is the only point off the diagonal, because the destination principle's residual distortion falls on consumers alone. Orig lies just inside. The customs union moves the platforms within the ellipse, from Dest to Cust, its intersection with the diagonal — the same profit for the platforms, higher welfare.

ecosystem can be larger, older or better connected than its partner; and the two sides of a data market are rarely equally placed, since the party that contributes the data often captures less of the value of each interaction than the party that uses it. This section asks who gains when the environment is unequal. Proposition 1 makes the question tractable, because each group's surplus is a function of its own participation alone, so "who gains" is answered by reading off participation levels.

We consider in turn an asymmetry across regions (Section 4.1) and an asymmetry across sides (Section 4.2). In each case we revisit the scenarios one by one and proceed as before: we first fix the crossing configuration, then solve for the fees under the two principle-based scenarios and for participation under monopoly, and finally check interiority. All proofs are gathered in Appendix B. Throughout the section, the parameters satisfy the following extension of Assumption 1; the surplus formulas of Proposition 1 carry over, since the derivation of $W^P = (n^P)^2$ and $W^C = (n^C)^2$ does not use the symmetry of the externality parameters.

Table 2: Numerical solutions for the five mechanisms when $\alpha = 0.20$ and $\gamma = 0.15$

	Destination	Origin	Monopoly	Customs union	First best
p^P	0.50	0.54	0.50	0.44	0
p^C	0.49	0.54	0.50	0.44	0
τ	0.11	-	-	0	0
$T = p^C + \tau$	0.60	0.54	0.50	0.44	0
n^P	0.73	0.70	0.77	0.86	1
n^C	0.66	0.70	0.77	0.86	1
Π per platform	0.76	0.76	0.77	0.76	0
W^P	0.53	0.49	0.59	0.74	1.70
W^C	0.43	0.49	0.59	0.74	1.70
W	2.48	2.52	2.72	3.00	3.40

Notes: Under the destination principle, the crossing fee is pinned down at $\tau = \gamma n^P$. Note that, by definition, $W = W^P + W^C + 2\Pi$ since Π is the per platform profit. Under the origin principle and under monopoly, the same player collects p^C and τ, so only the total T is determined and any split with $0 \leq \tau \leq \gamma n^P$ gives the same outcome. The first-best column reports the zero-fee implementation of Proposition 3; any common fee weakly below $\alpha + \gamma$ also sustains full participation, shifting surplus from participants to the platforms without changing W.

Extended notations:

- α_k^P, α_k^C: marginal value of a cross-side interaction with a participant from the same region (so far, we had $\alpha_1^P = \alpha_2^P = \alpha_1^C = \alpha_2^C$).
- γ_k^P, γ_k^C: marginal value of a cross-side interaction with a participant from the other region (so far, we had $\gamma_1^P = \gamma_2^P = \gamma_1^C = \gamma_2^C$).
- c: cost incurred by a data provider to join the platform (so far, we had $c = 0$).

Assumption 2 (Extension to asymmetric environments)**.** *For every region $k \in \{1,2\}$ and every side $s \in \{P, C\}$,*

$$0 \;<\; \gamma_k^s \;<\; \alpha_k^s \;\leq\; \tfrac{1}{5}, \qquad \textit{and} \qquad c \;\geq\; 0.$$

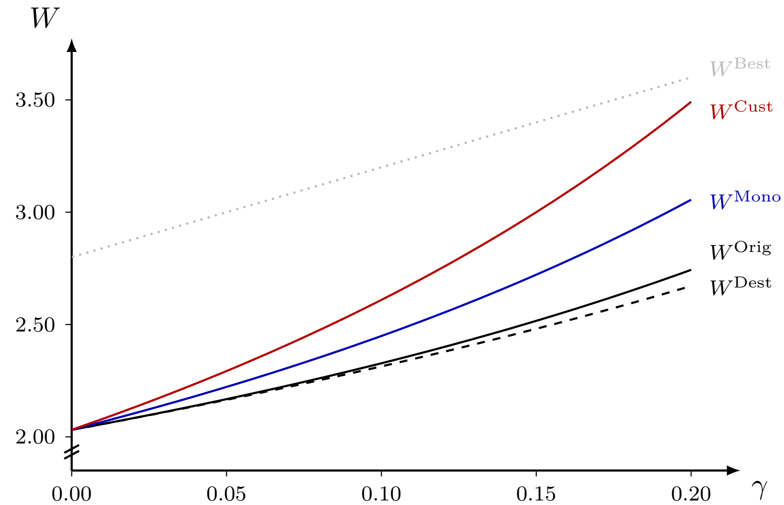
4.1 Distributional consequences across regions

Let region k's participants, on both sides, value a cross-region interaction at γ_k, possibly different from γ_l. This captures a newer or smaller ecosystem valuing access to an established one more than the reverse. The inversion (4) becomes

$$p_k^P = 1 - n_k^P + \alpha\, n_k^C + \gamma_k\, n_l^C, \qquad T_{kl}^C = 1 - n_k^C + \alpha\, n_k^P + \gamma_k\, n_l^P, \tag{13}$$

and the analysis proceeds as before. The condition under which region-k consumers cross becomes $\gamma_k\, n_l^P \geq \tau_{kl}^C$, and the crossing configuration is as in the symmetric model: in every scenario studied here, the consumers of both regions cross, for the reasons given in Section 2.1; under the destination principle the crossing fee τ_{lk}^C set by platform k is capped at $\gamma_l\, n_k^P$ and the cap binds.

Figure 3: Welfare under the five scenarios as a function of the inter-platform externality γ



Notes: Numerical results obtained for all $0 < \gamma < \alpha$ with $\alpha = 0.20$. At $\gamma = 0$ the two regions decouple and most scenarios coincide. Note that the welfare value at $\gamma = 0$ is positive: except for the social planner, it corresponds to the case of two regional monopolists without cross-region interactions.

Proposition 10 (Monopoly equalizes participants across regions)**.** *Under monopoly with $\gamma_k \neq \gamma_l$,*

$$n_k^P = n_k^C = n_l^P = n_l^C = \frac{1}{2 - 2\alpha - \gamma_1 - \gamma_2}, \qquad p_k^P = T_{kl}^C = \frac{1 - \alpha - \gamma_l}{2 - 2\alpha - \gamma_1 - \gamma_2}.$$

Participant surplus is therefore identical in the two regions, $\frac{1}{2}(n_k^P)^2 + \frac{1}{2}(n_k^C)^2 = \frac{1}{2}(n_l^P)^2 + \frac{1}{2}(n_l^C)^2$, and the entire asymmetry falls on platform rent:

$$\Pi_k - \Pi_l = \frac{2\,(\gamma_k - \gamma_l)}{\bigl(2 - 2\alpha - \gamma_1 - \gamma_2\bigr)^2}.$$

Proof. See Appendix B. □

Participation is equalized because the monopolist's profit depends on (γ_1, γ_2) only through their sum $\gamma_1 + \gamma_2$. The monopolist internalizes both ends of every cross-region interaction — the value to the consumer who crosses and the value to the provider who is reached — so what matters to it is the total value of an interaction, not how that value is split between the two regions. The split determines only where the rent is collected. The closed forms make this explicit: $p_k^P - p_l^P = (\gamma_k - \gamma_l)/(2 - 2\alpha - \gamma_1 - \gamma_2)$, so the region that values the link more is charged more — through the term $\gamma_k n_l^C$ of the inversion (13) — and yields more rent, yet its participants end up exactly as well off as the other region's. A single owner is, in this precise sense, even-handed between regions. The origin principle is not: each platform prices on its own γ_k, and participation diverges accordingly.

Proposition 11 (The origin principle transmits regional asymmetry)**.** *Under the origin principle with $\gamma_1 > \gamma_2$, participation is symmetric across the two sides within each region, $n_k^P = n_k^C =: n_k$, and the region that values the link more is charged more yet participates more:*

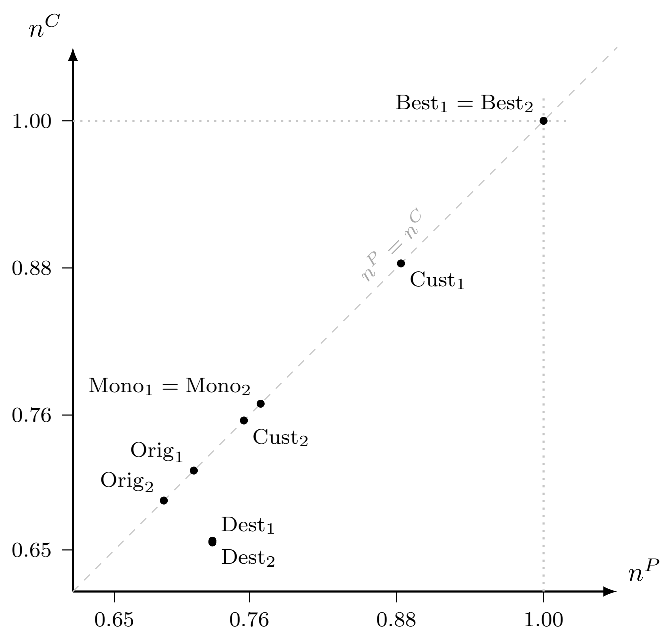
$$p_1^{P,\mathrm{Orig}} > p_2^{P,\mathrm{Orig}} \qquad \text{and} \qquad n_1^{\mathrm{Orig}} > n_2^{\mathrm{Orig}},$$

the participation gap being of first order in $\gamma_1 - \gamma_2$, so that region 1 obtains strictly higher participant surplus. Both regions participate, and gain, strictly less than under monopoly: $n_k^{\mathrm{Orig}} < n^{\mathrm{Mono}}$ for both k. Closed forms are given in Appendix B.

Proof. See Appendix B. □

Figure 4 and Table 3 show what Proposition 11 implies. In the participation space of the figure, each scenario appears once per region. Under the origin principle the two regions' points separate along the diagonal: the region that values the link more participates more, hence obtains more surplus. Under monopoly the two points coincide, and both lie further out than either origin point, so each region — the low-valuation one especially — is better off under a single owner. The transmission thus operates in favour of the region that values the link more; what the two regions share is the loss relative to monopoly, and the region that values the link less loses more. A data space whose members value the link unequally cannot expect uncoordinated fee-setting to treat them evenly.

Figure 4: Regional asymmetry in participation space



Notes: Numerical results for $\alpha = 0.2$, $\gamma_1 = 0.18$ and $\gamma_2 = 0.12$, the parameters of Table 3. Each scenario appears once per region, as (n_k^P, n_k^C). Monopoly and the first best equalize the two regions, so their points coincide; the origin principle spreads the regions along the diagonal; the destination principle leaves them nearly indistinguishable. The regional customs union of Proposition 14 (region-specific base fees, zero crossing fees) raises participation in both regions relative to the destination baseline; it pushes region 1, whose fees it cuts the most, far beyond the monopoly level, while region 2 lands slightly below it.

Each origin platform prices its own participants off the externality parameter that those participants themselves enjoy, γ_k, rather than off the total value $\gamma_1 + \gamma_2$ of a cross-region interaction. The asymmetry of the environment therefore passes through, at first order, into unequal fees and unequal participation: the region that values the link more is charged more, but the higher valuation more than compensates and it participates more. At $\gamma_1 = \gamma_2 = \gamma$ the closed forms collapse to Proposition 5.

The destination principle behaves very differently. Platform k keeps its three instruments $\left(p_k^P, p_k^C, \tau_{lk}^C\right)$, and the cap on the crossing fee binds, exactly as in the symmetric model.

Proposition 12 (The destination principle nearly equalizes across regions)**.** *Under the destination* *principle with $\gamma_1 \neq \gamma_2$, the provider fee satisfies*

$$p_k^{P,\text{Dest}} = \tfrac{1}{2}\Big[1 + (\gamma_k - \gamma_l)\, n_l^C\Big].$$

If $\gamma_1 > \gamma_2$, the region that values the link more participates slightly less, on both sides:

$$n_1^{P,\text{Dest}} \; < \; n_2^{P,\text{Dest}} \qquad \textit{and} \qquad n_1^{C,\text{Dest}} \; < \; n_2^{C,\text{Dest}},$$

both gaps being proportional to $\alpha\,(\gamma_1+\gamma_2)\,(\gamma_1-\gamma_2)$ and of opposite *sign to the gap under the origin principle. The exact expressions are given in Appendix B.*

Proof. See Appendix B. □

The binding crossing fee is what restores the monopolist's even-handedness. Because platform k can charge foreign crossers exactly the value $\gamma_l\, n_k^P$ they obtain, one more of its providers is worth to it the full value that this provider creates, at home and abroad: the provider-side equilibrium condition, $2n_k^P = 1 + 2\alpha\, n_k^C + (\gamma_1+\gamma_2)\, n_l^C$, is exactly the monopolist's, and like the monopolist's it depends on the regional valuations only through their sum. The residual asymmetry enters through the consumer side alone, and only at third order. The crossing revenue is where the asymmetry goes: platform 2 — in the region that values the link less — collects the larger crossing revenue, in proportion γ_1/γ_2 to what platform 1 collects, which is why the two platforms' profits end up nearly equal even though their participants value the link unequally.

Table 3: Regional asymmetry when region 1 values cross-region interactions more ($\gamma_1 > \gamma_2$)

	Destination	Origin	Monopoly	Customs union	First best
$p_1^P \;/\; p_2^P$	0.52 / 0.48	0.55 / 0.53	0.52 / 0.48	0.43 / 0.50	0 / 0
$T_1 \;/\; T_2$	0.62 / 0.58	0.55 / 0.53	0.52 / 0.48	0.43 / 0.50	0 / 0
$n_1^P \;/\; n_2^P$	0.73 / 0.73	0.71 / 0.69	0.77 / 0.77	0.88 / 0.76	1 / 1
$n_1^C \;/\; n_2^C$	0.66 / 0.66	0.71 / 0.69	0.77 / 0.77	0.88 / 0.76	1 / 1
$\Pi_1 \;/\; \Pi_2$	0.76 / 0.76	0.79 / 0.74	0.80 / 0.73	0.76 / 0.76	0 / 0
$W_1^C + W_1^P \;/\; W_2^C + W_2^P$	0.48 / 0.48	0.51 / 0.48	0.59 / 0.59	0.78 / 0.57	1.76 / 1.64
$W_1 \;/\; W_2$	1.24 / 1.24	1.30 / 1.21	1.40 / 1.33	1.54 / 1.33	1.76 / 1.64
W	2.48	2.51	2.72	2.87	3.40

Notes: Numerical results for $\alpha = 0.2$, $\gamma_1 = 0.18$ and $\gamma_2 = 0.12$. Under monopoly participation and participant surplus are equal across regions and the whole asymmetry falls on platform rent. The origin principle passes the asymmetry through into unequal participation, in favour of the region that values the link more; the destination principle transmits it only at third order, with the opposite sign. Regional welfare W_k is the participant surplus of region k plus Π_k, and $W = W_1 + W_2$. The customs-union column is the regional customs union of Proposition 14: region-specific base fees, zero crossing fees, so $T_k = p_k^C$; the symmetric customs union of Section 3.2 is infeasible at these parameters (Proposition 13). The first best is implemented by zero fees; every participant is then inframarginal, so regional surplus exceeds what the interior formulas would give.

Table 3 summarizes. The regional surplus gap under the destination principle is an order of magnitude smaller than under the origin principle, and of the opposite sign. A data space whose members value the link unequally should therefore not expect the two principle-based scenarios to treat its regions alike: only the origin principle transmits the asymmetry to a quantitatively meaningful extent, and it does so in favour of the region that values the link more, while both regions remain worse off than under a single owner.

How does the customs union fare across unequal regions? The acceptance constraint (12) becomes two constraints, $\Pi_k \geq \Pi_k^{\text{Dest}}$ for $k = 1, 2$, and feasibility is no longer automatic. The next two propositions treat in turn the customs union of Section 3.2, which is symmetric by construction — one base-fee pair common to the two regions, zero crossing fees — and a region-asymmetric variant that keeps the crossing fees at zero but lets the base fees differ across regions.

Proposition 13 (Feasibility of the symmetric customs union across regions)**.** *Consider the customs union of Section 3.2, proposed to platforms whose regions value the link unequally. At $\gamma_1 = \gamma_2$ it is feasible with slack: common monopoly fees leave each platform the margin $\Pi^{\text{Mono}} - \Pi^{\text{Dest}} > 0$, and feasibility therefore extends to small asymmetries. Under stronger asymmetry it can fail, the platform of the region that values the link less refusing every common fee pair: Example 1 gives an analytical case.*

Proof. See Appendix B. □

The reason feasibility is fragile is the crossing revenue highlighted after Proposition 12: under the destination baseline the crossing fee redistributes between the platforms, in favour of the platform whose region values the link less, and abolishing the fee abolishes the redistribution along with it.

One might have expected the opposite from Proposition 10: if the monopolist perfectly equalizes the two regions, should the two acceptance constraints not be ellipses with a common centre, as in Figure 2? They are still ellipses — each Π_k is a concave quadratic, so each set $\{\Pi_k \geq \Pi_k^{\text{Dest}}\}$ is an elliptical region in fee space — but they are *not* concentric. What the monopolist equalizes is the participants, not the platforms: its rent is split unequally, $\Pi_1 - \Pi_2 = 2(\gamma_1 - \gamma_2)\big(n^{\text{Mono}}\big)^2 > 0$ by Proposition 10, so the common monopoly fee pair is the centre of the *sum* $\Pi_1 + \Pi_2$, while each constraint is centred on its own platform's profit maximizer. The outside options, by contrast, are nearly equal, because the destination baseline's crossing fees redistribute. Feasibility of the symmetric customs union therefore requires the monopoly rent gap to be absorbed by the symmetric slack: roughly, $(\gamma_1 - \gamma_2)\big(n^{\text{Mono}}\big)^2 \lesssim \Pi^{\text{Mono}} - \Pi^{\text{Dest}}$, and since that slack is small, the intersection of the two ellipses above the outside options empties at mild asymmetry already.

Example 1 (An analytically infeasible customs union)**.** *Let $\alpha = \frac{1}{5}$, $\gamma_1 = \frac{3}{20}$ and $\gamma_2 = \frac{1}{20}$, a stronger asymmetry than in Table 3. Under free interconnection, platform 2 earns revenue from its own participants only, so whatever base fees — common or region-specific, positive or negative — the data space imposes*

$$\Pi_2 \;\leq\; \frac{\big(1+\gamma_2\big)^2}{2\,\big(1-\alpha\big)} \;=\; \frac{441}{640} \;=\; 0.6891 \;<\; 0.7095 \;=\; \Pi_2^{\text{Dest}},$$

so platform 2 refuses every customs union with $\tau_1 = \tau_2 = 0$. The bound uses only the inversion (13) and the fact that region-1 participation cannot exceed the unit mass; it is derived in Appendix B, where Π_2^{Dest} is evaluated on the closed-form system of Proposition 12.

Proposition 14 (Regional customs union)**.** *Let the data space instead propose region-specific base-fee pairs, keeping the crossing fees at zero. (i) The feasible set of this regional customs union contains that of the symmetric one, so it is feasible whenever the latter is, and attains weakly higher welfare. (ii) Free interconnection itself still caps the profit of the platform whose region values the link less: with $\gamma_2 < \gamma_1$, feasibility requires*

$$\big(1+\gamma_2\big)^2 \;\geq\; 2\,(1-\alpha)\,\Pi_2^{\text{Dest}},$$

so the regional customs union too becomes infeasible under sufficiently strong asymmetry.

Proof. See Appendix B. □

For moderate asymmetry the regional customs union is feasible, and Table 3 and Figure 4 report the welfare-maximizing one. Two features stand out. First, it leaves both platforms exactly at their outside options and delivers a welfare level well above monopoly, just as the symmetric customs union does in the symmetric model. Second, unlike the monopolist, it does not equalize the regions: it sets the *lower* base fees in the region that values the link more, expanding that region's participation the furthest, because an extra participant there generates more cross-region value per unit of platform rent sacrificed. Deprived of the crossing fee as a transfer instrument, in other words, the data space buys welfare where it is cheapest, and the asymmetry of valuations reappears as an asymmetry of treatment.

4.2 Distributional consequences across sides

Now let the two sides value a match differently: providers value same- and cross-region interactions at α^P, γ^P, consumers at α^C, γ^C, and write $\Sigma := \alpha^P + \gamma^P + \alpha^C + \gamma^C$ for the total externality intensity. This is the empirically relevant case for data sharing, where the side that supplies the data frequently captures less of the value of each match than the side that consumes it. The inversion becomes

$$p^P = 1 - n^P + (\alpha^P + \gamma^P)\, n^C, \qquad T = 1 - n^C + (\alpha^C + \gamma^C)\, n^P, \tag{14}$$

and each side is still priced according to the externality intensity that it enjoys — throughout this section, a side's "own" externality parameters refer to the value its members derive from the presence of the other side, not to the value they create for it, as (14) makes explicit. Profit in participation space is

$$\Pi(n^P, n^C) = n^P + n^C - (n^P)^2 - (n^C)^2 + \Sigma\, n^P n^C. \tag{15}$$

Proposition 1 is unchanged: $W^P = (n^P)^2$, $W^C = (n^C)^2$. Region-k consumers now cross whenever $\gamma^C n_l^P \geq \tau_{kl}^C$, and, as before, every scenario studied operates in the configuration where the consumers of both regions cross.

Proposition 15 (Monopoly equalizes participation across the two sides)**.** *Under monopoly, $n^P = n^C = 1/(2 - \Sigma)$ for any $\alpha^P, \gamma^P, \alpha^C, \gamma^C$, where $\Sigma := \alpha^P + \gamma^P + \alpha^C + \gamma^C$. Providers and consumers therefore obtain exactly equal surplus, however unequally they value a match. Only a side-specific participation cost breaks this: if providers bear a fixed cost $c \geq 0$ of joining, then $n^{P,\text{Mono}} = \big(2(1 - c) + \Sigma\big)/\big(4 - \Sigma^2\big)$ and*

$$n^P - n^C = -\frac{c}{2 + \Sigma}.$$

Proof. See Appendix B. □

The four externality parameters enter only through their sum Σ, for the reason behind Proposition 10, one level down: the monopolist internalizes both ends of every match, so all it needs to know is what a match is worth in total; how that total is split between the two sides affects only which side it collects the money from. The fees are asymmetric — the side that enjoys more of the value is charged more — but participation, hence surplus, is the same on both sides, and only a side-specific participation cost can separate them.

Under the origin principle, by contrast, $n^P \neq n^C$ as soon as the two sides differ in their valuation of cross-side interactions, and the side that values a match less ends up with lower participation and lower surplus. The next two propositions establish this analytically, one per principle-based scenario; Table 4 illustrates.

Proposition 16 (The origin principle transmits side asymmetry)**.** *Under the origin principle, the equilibrium is symmetric across regions, and the two sides' participation levels admit closed forms, given in Appendix B. In particular:*

(i) if the two sides differ only in the same-region parameter ($\gamma^P = \gamma^C$), the side with the larger α participates more, the gap being of second order in the externality parameters;

(ii) if they differ only in the cross-region parameter ($\alpha^P = \alpha^C$), the side with the larger γ participates more, the gap being of first order;

(iii) if one side values both interactions weakly more, and the cross-region one strictly more, then that side participates strictly more and obtains strictly higher surplus.

Proof. See Appendix B. □

Under the origin principle each platform prices each side off the externality that side enjoys, so the asymmetry of the environment passes through into the asymmetry of outcomes. The destination principle behaves very differently: it does not transmit the asymmetry of the environment so much as impose an asymmetry of its own, and always in the same direction. Platform k keeps its three instruments and its cap on the crossing fee, $\tau^C_{lk} \leq \gamma^C n^P_k$, binds, exactly as in the symmetric model.

Proposition 17 (The destination principle favours the provider side)**.** *Under the destination principle, the equilibrium is symmetric across regions and satisfies the monopolist's provider-side first-order condition exactly,*

$$2\, n^{P,\text{Dest}} = 1 + \Sigma\, n^{C,\text{Dest}}, \qquad \textit{with provider fee} \qquad p^{P,\text{Dest}} = \tfrac{1}{2}\Big[1 + \big(\alpha^P - \alpha^C + \gamma^P - \gamma^C\big)\, n^{C,\text{Dest}}\Big],$$

where $\Sigma := \alpha^P + \gamma^P + \alpha^C + \gamma^C$. Whatever the split of the value of a match between the two sides, $n^{P,\text{Dest}} > n^{C,\text{Dest}}$: providers participate more, and obtain strictly higher surplus, than consumers. The consumer-side condition, and the inequality behind this comparison, are given in Appendix B.

Proof. See Appendix B. □

This generalizes the side asymmetry found under symmetric parameters in Proposition 2, and identifies its source. Through the binding cap, the platform that levies the crossing fee captures the whole value that one more of its providers creates: α^C per own-region consumer, through the base fee it can then charge them, and γ^C per foreign crosser, through the crossing fee. On the provider side it therefore behaves exactly like a monopolist owning both platforms — whence the monopolist's provider-side condition, and a provider fee that, with a provider cost c, still absorbs exactly half of it. On the consumer side it cannot do the same: the cross-region value $\gamma^C n^P_l$ obtained by its own consumers has already been extracted in full by the rival's crossing fee, so the home platform cannot price against it. The double marginalization identified in Section 2.3 is thus, in full generality, a consumer-side phenomenon: whatever the split of the value of a match, the consumer side is the one left behind.

Table 4 provides a numerical illustration with parameters under which consumers value a match roughly twice as intensely as providers. Note the reversal: the provider side does better under the destination principle than under the origin principle ($W^P = 0.41$ against 0.37) even though total welfare is lower, because the destination principle prices providers as a monopolist would and loads its entire distortion onto consumers. A policy maker whose objective is provider participation should therefore not expect the move from the destination to the origin principle to help that side; the instrument that helps is a price constraint, to which we return below.

Table 4: Side asymmetry when consumers value interactions more ($\alpha^C > \alpha^P$ and $\gamma^C > \gamma^P$)

	Destination	Origin	Monopoly	Customs union max W	Customs union max W^P	First best max W	First best max W^P
n^P	0.64	0.61	0.66	0.71	0.72	1	1
n^C	0.59	0.63	0.66	0.71	0.67	1	1
p^P	0.45	0.49	0.44	0.40	0.38	0	0
T	0.62	0.57	0.56	0.52	0.57	0	0
W^P	0.41	0.37	0.43	0.50	0.52	1.30	1.30
W^C	0.35	0.40	0.43	0.50	0.45	1.66	1.66
Π per platform	0.65	0.66	0.66	0.65	0.65	0	0
W	2.07	2.08	2.18	2.31	2.28	2.96	2.96

Notes: Numerical results for $\alpha^P = 0.10$, $\gamma^P = 0.05$, $\alpha^C = 0.18$, $\gamma^C = 0.15$. Both customs-union columns leave the platforms exactly at their outside option Π^{Dest} and differ only in the objective (total welfare versus provider surplus). Since Π is per platform, $W = 2\Pi + W^P + W^C$. The first best is implemented by zero fees; every participant is then inframarginal, so side surplus exceeds what the interior formulas would give. With nonnegative fees, the provider-optimal first best coincides with the total-welfare first best: zero fees maximize participation on both sides, and any positive fee lowers provider surplus either directly or through consumer participation, so the two first-best columns are identical. (Allowing subsidies unbacked by any revenue would make W^P unbounded, so fees are restricted to be nonnegative.)

Figure 5 presents the side-asymmetric problem in participation space; it is the analogue of Figure 2. The platforms' participation constraint is again an ellipse, but the objective is no longer symmetric, so the relevant optima are no longer where the ellipse meets the diagonal. Because $W^P = (n^P)^2$ depends on n^P alone, the iso-provider-welfare curves are vertical lines, and maximizing provider surplus means moving to the rightmost point of the ellipse — denoted Cust^P — rather than to the total-welfare point Cust.

The pattern of Section 4.1 thus repeats one level down, with one refinement. Centralization equalizes, because a single owner internalizes both ends of every match and cares only about its total value; how that value is split across regions or across sides determines where the rent is collected, never who participates. The origin principle transmits the asymmetry of the environment into an asymmetry of outcomes, across regions as across sides, because each platform prices each group off the share of the value that this group happens to enjoy. The destination principle sits in between: its binding crossing fee makes each platform price providers exactly as a monopolist would, so it inherits the monopolist's even-handedness across regions almost fully — but across sides it is systematically partial to providers, because its residual distortion falls on consumers alone.

Provider participation costs. Let a provider bear a fixed cost $c \geq 0$ of joining, so that $U_k^P = v^P - c + \alpha^P n_k^C + \gamma^P n_{lk}^C - p_k^P$. The inversion (14) becomes

$$p^P = 1 - c - n^P + (\alpha^P + \gamma^P)\, n^C, \qquad T = 1 - n^C + (\alpha^C + \gamma^C)\, n^P, \tag{16}$$

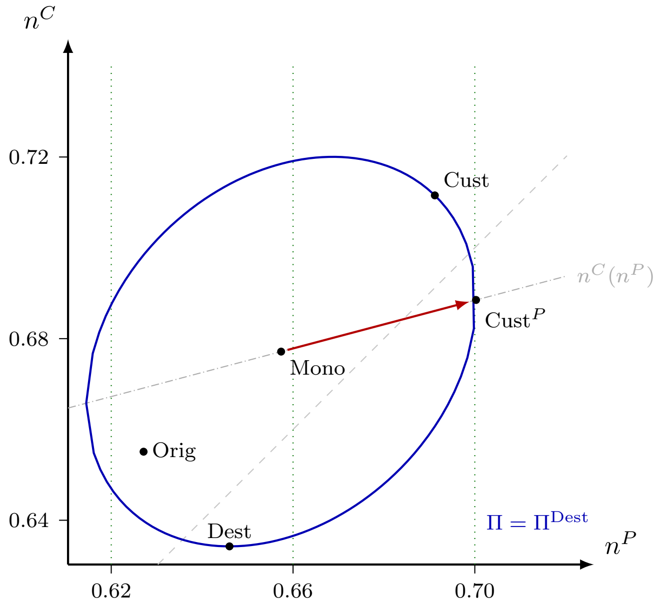
so the cost is a pure provider-side shifter, and

$$\Pi(n^P, n^C) = \left(n^P + n^C\right) - c\, n^P - (n^P)^2 - (n^C)^2 + \Sigma\, n^P n^C. \tag{17}$$

The monopolist's first-order conditions $\partial\Pi/\partial n^P = \partial\Pi/\partial n^C = 0$ give

$$n^{P,\text{Mono}} = \frac{2 - 2c + \Sigma}{4 - \Sigma^2}, \qquad n^{C,\text{Mono}} = \frac{1 + \Sigma\, n^{P,\text{Mono}}}{2}, \qquad n^{P,\text{Mono}} - n^{C,\text{Mono}} = -\frac{c}{2 + \Sigma}, \tag{18}$$

Figure 5: Cross-side asymmetry in participation space



Notes: Numerical results for $\alpha^P = 0.15$, $\gamma^P = 0.05$, $\alpha^C = 0.25$, $\gamma^C = 0.08$ and $c = 0.05$, so that consumers value a match more intensely than providers. The blue curve is the platforms' participation constraint $\Pi = \Pi^{\text{Dest}}$. Iso-provider-welfare loci are vertical, so the provider-optimal constraint Cust^P is the rightmost point of the ellipse, distinct from the total-welfare point Cust; the dash-dotted line is the ridge $n^C(n^P) = (1 + \Sigma n^P)/2$ along which Π is maximized for each n^P.

which reproduces the closed forms of Proposition 15 (whose full proof, including the configuration and symmetry arguments, is in Appendix B): at $c = 0$ the two sides are equalized at $1/(2 - \Sigma)$ whatever the four externality parameters, and the gap between them is generated by the cost alone, not by the difference in how the two sides value a match. Since p^P carries the term $-c$, the equilibrium provider fee can be negative when c is large relative to $\alpha^P + \gamma^P$: the platform subsidizes providers in order to harvest the resulting consumer-side revenue.

If instead the cost varies by region, $c_1 \neq c_2$, with the two sides symmetric, then under monopoly

$$p_k^P = \frac{1 - c_k}{2}, \qquad T_{kl}^C = \tfrac{1}{2}, \qquad k = 1, 2, \tag{19}$$

so the monopolist absorbs exactly half of each region's own cost, region by region, and leaves the consumer-side price entirely untouched. The pass-through is the familiar 50% of linear-demand monopoly; what is specific here is that it is entirely local — region k's price responds to c_k alone — and entirely confined to the cost-bearing side.

When is the zero-fee benchmark still the first best, and when do subsidies become part of efficient pricing? The next proposition sorts the possible regimes by the size of the cost, and relates the first best's fees to the monopolist's.

Proposition 18 (Participation costs, subsidies, and the first best)**.** *Let providers bear a participation cost $c \geq 0$, and write $s^P := \alpha^P + \gamma^P$ and $s^C := \alpha^C + \gamma^C$ for the externality intensity each side enjoys, $\Sigma = s^P + s^C$.*

(i) The first best has full participation on both sides if and only if $c \leq \Sigma$; for $c > \Sigma$ it keeps $n^C = 1$ but reduces provider participation to $n^P = 1 - c + \Sigma < 1$.

(ii) Zero fees implement the first best if and only if $c \leq s^P$. Beyond that threshold the first-best provider fee is negative: when full participation is still efficient ($s^P < c \leq \Sigma$), any $p^P \leq s^P - c < 0$ implements it; when it no longer is ($c > \Sigma$), the supporting fee is exactly $p^P = -s^C$, the externality one provider creates.

(iii) The monopolist subsidizes providers, $p^{P,\text{Mono}} < 0$, if and only if $s^C > s^P$ and $c > c^* := 1 - \dfrac{s^C - s^P}{2 - \Sigma\, s^C}$. Under Assumption 2, $s^P \leq \Sigma < c^*$: whenever the monopolist subsidizes, the first best requires a subsidy as well and involves less than full provider participation — but not conversely, since for $s^P < c \leq c^*$ the first best subsidizes while the monopolist still charges a positive fee.

Proof. See Appendix B. □

At the parameters of Table 5, $s^P = 0.20$ and $\Sigma = 0.53$: any cost between the two — say $c = 0.3$ — makes zero fees inefficient while full participation remains efficient, and implementing the first best then requires subsidizing providers by at least $c - s^P$. The intuition for the ordering in (iii) is that the planner internalizes the whole externality s^C that a provider creates, while the monopolist internalizes only the revenue it can extract from it, so the planner reaches for the subsidy at a much lower cost than the monopolist does.

The possibility of negative provider fees admits a tractable closed form, and an example in which it arises.

Example 2 (Negative provider fees and inefficient participation)**.** *Under monopoly the provider fee turns negative exactly when a cost-burdened provider side contributes much more value than it enjoys; the closed form of the fee, the sign threshold, and the interiority checks are in Appendix B. At $\alpha^P = \frac{1}{20}$, $\gamma^P = \frac{1}{40}$, $\alpha^C = \frac{1}{5}$, $\gamma^C = \frac{3}{20}$, the fee is negative for every participation cost $c \gtrsim 0.85$: at $c = \frac{9}{10}$ the monopolist charges providers $p^P \approx -0.02$ — it pays them to join — and recoups the subsidy on the consumer side, with $T \approx 0.52$ and a profit of ≈ 0.28 per platform. Note that the subsidy does not come close to inducing full participation: $n^P \approx 0.16$ against $n^C \approx 0.54$. The sign of the fee changes; the interior logic of participation does not.*

The provider-welfare-maximizing customs union. None of the above tells a data space what it should do, only what each rule delivers. Suppose then that the orchestrator's objective is not total welfare but the welfare of one side — e.g., because data providers are small firms, or farmers, whose participation is the policy objective. By Proposition 1, maximizing provider surplus is exactly maximizing n^P subject to the platforms' participation constraint (12), and the problem has a closed-form solution. For any target n^P, the n^C that most relaxes the constraint is the one maximizing $\Pi(n^P, \cdot)$, namely

$$n^C(n^P) = \frac{1 + \Sigma\, n^P}{2}, \tag{20}$$

which is the monopolist's own best response: when the objective is providers alone, consumer participation is purely instrumental, valued only for the profit it generates to finance a further push on n^P. Substituting into (17) gives the frontier

$$\Pi_{\max}(n^P) = \Big(\frac{\Sigma^2}{4} - 1\Big)(n^P)^2 + \Big(1 + \frac{\Sigma}{2} - c\Big)n^P + \frac{1}{4}, \tag{21}$$

a downward-opening parabola since $\Sigma < 2$.

Proposition 19 (Provider-welfare-maximizing price constraint)**.** *The provider-welfare-maximizing customs union sets n^P at the larger root of*

$$\big(\Sigma^2-4\big)(n^P)^2+\big(4+2\Sigma-4c\big)n^P+\big(1-4\Pi^{\mathrm{Dest}}\big)=0,$$

with n^C from (20) *and prices from* (16)*. This delivers the highest provider surplus attainable by any mechanism respecting the platforms' participation constraint, strictly above monopoly and above the total-welfare-maximizing customs union of Proposition 8.*

Proof. See Appendix B. □

Table 5: Side asymmetry when providers incur a cost to participate

	Destination	Origin	Monopoly	Customs union max W	Customs union max W^P	First best
n^P	0.64	0.62	0.65	0.69	0.70	1
n^C	0.63	0.65	0.67	0.71	0.68	1
p^P	0.43	0.46	0.43	0.40	0.39	0
T	0.58	0.55	0.54	0.52	0.55	0
Π per platform	0.65	0.65	0.65	0.65	0.65	0
W^P	0.41	0.39	0.43	0.47	**0.48**	1.30
W^C	0.40	0.42	0.45	**0.50**	0.47	1.66
W	2.10	2.10	2.17	**2.26**	2.24	2.96

Notes: Numerical results for $\alpha^P=0.15$, $\gamma^P=0.05$, $\alpha^C=0.25$, $\gamma^C=0.08$ and $c=0.05$. Consumers value a match more intensely than providers. Both customs-union columns leave the platforms exactly at their outside option Π^{Dest}; they differ only in how the surplus is split between the two sides. Bold marks the column that maximises each row among the implementable mechanisms. The first best is implemented by zero fees; every participant is then inframarginal, so side surplus exceeds what the interior formulas would give, and $W=2\Pi+W^P+W^C$ since Π is per platform. At these parameters full participation is efficient despite the provider cost, because $c<\Sigma$: the externality a marginal provider creates exceeds their cost. For $c>\Sigma$ the first best would have interior provider participation $n^P=1-c+\Sigma$, but implementing it requires a provider *subsidy* $p^P=-(\alpha^C+\gamma^C)$ — the externality each provider creates — so no cost makes the first best both interior and supported by nonnegative fees (Proposition 18).

Table 5 illustrates, with providers calibrated to value interactions less than consumers and to bear a small cost. Both customs-union columns leave the platforms exactly at their outside option; they differ only in how the resulting surplus is split between the two sides.

5 Discussion and conclusion

Data spaces are based on interoperability, but we argue that interoperability alone is not enough. In the destination-principle baseline of Section 2.3, platforms are technically connected, and they independently set prices for their own participant base. That scenario is the worst of the five we study, in terms of welfare and profit.

One may think that forbidding a separate charge for crossing the border would improve this outcome. This turns out to be equivalent to the origin-principle governance mechanism analyzed in Proposition 5. We show that it does, indeed, increase both welfare and profit relative to the destination principle. However, it still does not outperform the monopoly benchmark.

Coordination on the base fees themselves can reduce the gap with the first best. The customs union of Section 3.2 constrains those fees subject only to each platform preferring the rule to its outside option, and it beats every other scenario, monopoly included. The reason is visible in Figure 2: the platforms' participation constraint traces an ellipse, the decentralized baseline sits on that ellipse, and the customs union simply moves along it to the point that maximizes participation — the platforms end up exactly as well off as they would have been alone, and everyone else better off. But it does require a data space to treat pricing as within its remit, which may be more ambitious than just establishing standards.

Distributional implications, not just efficiency comparisons, matter for governance choices. In that respect, the difference between a monopolist and the two decentralized mechanisms considered is striking. A monopolist owning both platforms equalizes: it leaves the two regions' participants exactly equally well off no matter how they value the connection, absorbing the whole asymmetry into where it collects its rent, and it equalizes the two sides of the market no matter how they value a match. Both decentralized principles instead pass the asymmetry of the environment straight through into unequal participation and unequal surplus. A customs union may, depending on the prices it proposes, deliver higher welfare than the monopolist and weakly Pareto-dominate the decentralized mechanisms. However, a customs union based on a single price per side across regions may not be feasible if the asymmetry between the platforms is too strong.

This paper is subject to several limitations. First, all consumers of a region are identical, so crossing is all-or-nothing and the crossing fee is a pure extraction instrument; with heterogeneous willingness to cross, the fee would trade extraction against the crossing margin and the full-extraction logic of Remark 2 would soften. Second, the model is static, has two regions, and abstracts from data quality, privacy and entry, all of which bear on the governance choices we study. Third, participants in data-sharing — typically firms — are often part of the same value chain as those they interact with on a data space; our model examines data-sharing in isolation of these other business relations.

References

Anderson, Simon P., Øystein Foros, and Hans Jarle Kind (Feb. 1, 2018). "Competition for Advertisers and for Viewers in Media Markets". In: *The Economic Journal* 128.608, pp. 34–54. DOI: 10.1111/ecoj.12428.

Armstrong, Mark (2006). "Competition in Two-Sided Markets". In: *The RAND Journal of Economics* 37.3, pp. 668–691.

Bakos, Yannis and Hanna Halaburda (2020). "Platform Competition with Multihoming on Both Sides: Subsidize or Not?" In: *Management Science* 66.12, pp. 5599–5607.

Belleflamme, Paul and Martin Peitz (2019). "Managing competition on a two-sided platform". In: *Journal of Economics & Management Strategy* 28.1, pp. 5–22. DOI: 10.1111/jems.12311.

Bergemann, Dirk, Alessandro Bonatti, and Alex Smolin (Jan. 2018). "The Design and Price of Information". In: *American Economic Review* 108.1, pp. 1–48. DOI: 10.1257/aer.20161079.

Caillaud, Bernard and Bruno Jullien (2003). "Chicken & Egg: Competition among Intermediation Service Providers". In: *The RAND Journal of Economics* 34.2, pp. 309–328. DOI: 10.2307/1593720.

Carballa-Smichowski, Bruno et al. (2026). "Inter-platform ecosystems". In: *Strategic Management Journal* 47.7, pp. 1840–1877. DOI: 10.1002/smj.70070.

Cornière, Alexandre de and Greg Taylor (Sept. 2024). "Data-Driven Mergers". In: *Management Science* 70.9, pp. 6473–6482. DOI: 10.1287/mnsc.2023.04104.

Correia-da-Silva, Joao et al. (2019). "Horizontal mergers between multisided platforms: Insights from Cournot competition". In: *Journal of Economics & Management Strategy* 28.1, pp. 109–124. DOI: 10.1111/jems.12309.

Ekmekci, Mehmet, Alexander White, and Lingxuan Wu (Oct. 2025). "Platform Competition and Interoperability: The Net Fee Model". In: *Management Science* 71.10, pp. 8842–8864. DOI: 10.1287/mnsc.2023.02810.

European Commission. Directorate General for Competition. (2019). *Competition policy for the digital era.* LU: Publications Office. DOI: 10.2763/407537.

Eustache, Lucas, Eric Brousseau, and Joëlle Toledano (Dec. 14, 2025). "The Economics of Data-Sharing: An Empirical Investigation of Data Sharing Ecosystems". In: *ICIS 2025 Proceedings*.

Evans, D. S. and M. D. Noel (Sept. 1, 2008). "The Analysis of Mergers That Involve Multisided Platform Businesses". In: *Journal of Competition Law and Economics* 4.3, pp. 663–695. DOI: 10.1093/joclec/nhn017.

Farboodi, Maryam and Laura Veldkamp (Feb. 2021). *A Model of the Data Economy*. w28427. Cambridge, MA: National Bureau of Economic Research, w28427. DOI: 10.3386/w28427.

Hagiu, Andrei (2009). "Two-Sided Platforms: Product Variety and Pricing Structures". In: *Journal of Economics & Management Strategy* 18.4, pp. 1011–1043. DOI: 10.1111/j.1530-9134.2009.00236.x.

Hutterer, Andreas, Barbara Krumay, and Manuel Muehlburger (Aug. 10, 2023). "What Constitutes a Dataspace? Conceptual Clarity beyond Technical Aspects". In: *AMCIS 2023 Proceedings*.

Jacobides, Michael G., Carmelo Cennamo, and Annabelle Gawer (2018). "Towards a theory of ecosystems". In: *Strategic Management Journal* 39.8, pp. 2255–2276. DOI: 10.1002/smj.2904.

— (Jan. 1, 2024). "Externalities and complementarities in platforms and ecosystems: From structural solutions to endogenous failures". In: *Research Policy* 53.1, p. 104906. DOI: 10.1016/j.respol.2023.104906.

Jones, Charles I. and Christopher Tonetti (Sept. 2020). "Nonrivalry and the Economics of Data". In: *American Economic Review* 110.9, pp. 2819–2858. DOI: 10.1257/aer.20191330.

Kerber, Wolfgang and Heike Schweitzer (2017). "Interoperability in the Digital Economy". In: *Journal of Intellectual Property, Information Technology and Electronic Commerce Law* 8, p. 39.

Kim, Jeong-Yoo (2026). "Data Portability and Interoperability Between Digital Platforms". In: *Journal of Economics & Management Strategy* 35.2, pp. 219–232. DOI: 10.1111/jems.12643.

Krämer, Jan and Shiva Shekhar (Mar. 1, 2025). "Regulating Digital Platform Ecosystems Through Data sharing and Data Siloing: Consequences for Innovation and Welfare". In: *Management Information Systems Quarterly* 49.1, pp. 123–154. DOI: 10.25300/MISQ/2024/18428.

Laatikainen, Gabriella and Tuomo Tuikka (Dec. 17, 2025). "Cross-Data Space Interoperability: Key Challenges". In.

Lin, Xiaogang et al. (Sept. 2025). "Mergers Between On-Demand Service Platforms: The Impact on Consumer Surplus and Labor Welfare". In: *Information Systems Research* 36.3, pp. 1612–1630. DOI: 10.1287/isre.2022.0553.

Martens, Bertin (Feb. 19, 2024). *An institutional economics approach to data spaces as data market intermediaries*. Rochester, NY. DOI: 10.2139/ssrn.4731126.

Motta, Massimo and Martin Peitz (Mar. 1, 2021). "Big tech mergers". In: *Information Economics and Policy*. Antitrust in the Digital Economy 54, p. 100868. DOI: 10.1016/j.infoecopol.2020.100868.

— (Apr. 15, 2024). "Denial of interoperability and future first-party entry". In: *International Journal of Industrial Organization*, p. 103070. DOI: 10.1016/j.ijindorg.2024.103070.

Parker, Geoffrey G., Marshall W. Van Alstyne, and Sangeet Paul Choudary (Mar. 28, 2016). *Platform Revolution: How Networked Markets Are Transforming the Economy and How to Make Them Work for You*. W. W. Norton & Company. 433 pp.

Prüfer, Jens and Christoph Schottmüller (2021). "Competing with Big Data". In: *The Journal of Industrial Economics* 69.4, pp. 967–1008. DOI: 10.1111/joie.12259.

Richter, Heiko (May 1, 2023). "Looking at the Data Governance Act and Beyond: How to Better Integrate Data Intermediaries in the Market Order for Data Sharing". In: *GRUR International* 72.5, pp. 458–470. DOI: 10.1093/grurint/ikad014.

Rochet, Jean-Charles and Jean Tirole (June 1, 2003). "Platform Competition in Two-Sided Markets". In: *Journal of the European Economic Association* 1.4, pp. 990–1029. DOI: 10.1162/154247603322493212.

Rysman, Marc (Sept. 2009). "The Economics of Two-Sided Markets". In: *Journal of Economic Perspectives* 23.3, pp. 125–143. DOI: 10.1257/jep.23.3.125.

Sandrini, Luca and Robert Somogyi (2023). "News Media Bargaining Codes". In: *SSRN Electronic Journal*. DOI: 10.2139/ssrn.4398480.

Teh, Tat-How et al. (2023). "Multihoming and Oligopolistic Platform Competition". In: *American Economic Journal: Microeconomics* 15.4, pp. 68–113.

Veldkamp, Laura and Cindy Chung (June 1, 2024). "Data and the Aggregate Economy". In: *Journal of Economic Literature* 62.2, pp. 458–484. DOI: 10.1257/jel.20221580.

Wysel, Matthew, Derek Baker, and William Billingsley (Oct. 1, 2021). "Data sharing platforms: How value is created from agricultural data". In: *Agricultural Systems* 193, p. 103241. DOI: 10.1016/j.agsy.2021.103241.

A Matrix-based analysis of the participation system

The system for a given crossing configuration

Let $x = (x_1, x_2) \in \{0,1\}^2$ record which regions cross, as in the proof of Proposition 4, and let T_k be the total fee paid by a participating region-k consumer. Every participant whose stand-alone value exceeds the relevant threshold participates, and stand-alone values are uniform on $[0,1]$ with unit mass on each side of each region, so a participation vector $n = (n_1^C, n_2^C, n_1^P, n_2^P)$ is an equilibrium of the second stage if and only if

$$n = \Big[\,\mathbf{1} - p + \mathcal{B}_x\, n\,\Big]_0^1, \qquad p := \big(T_1, T_2, p_1^P, p_2^P\big), \tag{22}$$

where $[\,\cdot\,]_0^1$ truncates each coordinate to $[0,1]$ and $\mathcal{B}_x$ collects the externality coefficients of (33). The map on the right of (22) is Lipschitz in the supremum norm with constant $\|\mathcal{B}_x\|_\infty \le \alpha + \gamma$, since truncation is 1-Lipschitz and each row of $\mathcal{B}_x$ has nonnegative entries summing to at most $\alpha + \gamma$. Under Assumption 1, $\alpha + \gamma < 2\alpha \le \frac{2}{5} < 1$, so the map is a contraction: for each fee vector the participation equilibrium exists, is unique, and is reached from any starting point. Whenever the solution of (22) is interior the truncation is inactive and the system is the linear one below; interiority is checked separately in each proof.

The system when consumers of both regions multi-home

With the unknowns ordered $\left(n_1^C, n_2^C, n_1^P, n_2^P\right)$, the case $x=(1,1)$ of (2)–(3) is $M\,n = \mathbf{1} - p$ with

$$M = \begin{pmatrix} 1 & 0 & -\alpha & -\gamma \\ 0 & 1 & -\gamma & -\alpha \\ -\alpha & -\gamma & 1 & 0 \\ -\gamma & -\alpha & 0 & 1 \end{pmatrix} = \begin{pmatrix} I & -E \\ -E & I \end{pmatrix}, \qquad E := \begin{pmatrix} \alpha & \gamma \\ \gamma & \alpha \end{pmatrix}, \tag{23}$$

which is (4) read from right to left. Write $\sigma := \alpha + \gamma$ and $\delta := \alpha - \gamma$ for the two eigenvalues of E, with eigenvectors $(1,1)$ and $(1,-1)$. Then

$$\det M = \det\left(I - E^2\right) = \det(I-E)\det(I+E) = (1-\sigma)(1+\sigma)(1-\delta)(1+\delta) = \left(1-\sigma^2\right)\left(1-\delta^2\right),$$

and the eigenvalues of M are $1 \pm \sigma$ and $1 \pm \delta$. Under Assumption 1 we have $0 < \delta < \sigma < 2\alpha \leq \frac{2}{5}$, so all four are strictly positive: M is symmetric positive definite, $\det M > 0$, and the map between prices and participation is a bijection.

The inverse

Since E is symmetric, M^{-1} is obtained blockwise as

$$M^{-1} = \begin{pmatrix} \left(I - E^2\right)^{-1} & \left(I - E^2\right)^{-1} E \\ \left(I - E^2\right)^{-1} E & \left(I - E^2\right)^{-1} \end{pmatrix}. \tag{24}$$

The matrices $\left(I - E^2\right)^{-1}$ and $\left(I - E^2\right)^{-1} E$ are diagonalized by the same two eigenvectors, with eigenvalues $\left(1-\sigma^2\right)^{-1}$, $\left(1-\delta^2\right)^{-1}$ and $\sigma\left(1-\sigma^2\right)^{-1}$, $\delta\left(1-\delta^2\right)^{-1}$ respectively, so that, writing $\det M = \left(1-\sigma^2\right)\left(1-\delta^2\right)$,

$$M^{-1} = \frac{1}{\det M} \begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{pmatrix}, \qquad \begin{aligned} A &= 1 - \alpha^2 - \gamma^2, & C &= \alpha\left(1 - \alpha^2 + \gamma^2\right), \\ B &= 2\alpha\gamma, & D &= \gamma\left(1 + \alpha^2 - \gamma^2\right). \end{aligned} \tag{25}$$

All four are strictly positive under Assumption 1. Two consequences are used in the proofs.

(i) Every fee lowers every participation level. Since $n = M^{-1}(\mathbf{1} - p)$ and M^{-1} has strictly positive entries, $\partial n_i / \partial p_j < 0$ for every pair (i,j). Equivalently, raising any intercept raises every participation level: if two fee vectors satisfy $p \leq p'$ coordinatewise, then $n(p) \geq n(p')$ coordinatewise. The same holds for any configuration x, because $\mathcal{B}_x$ is nonnegative with spectral radius below one, so $\left(I - \mathcal{B}_x\right)^{-1} = \sum_{m \geq 0} \mathcal{B}_x^m$ is nonnegative.

(ii) Switching a region from single-homing to multi-homing raises every participation level. Passing from $x_k = 0$ to $x_k = 1$ at unchanged $\left(p_1^P, p_2^P, T_1, T_2\right)$ adds the term γn_l^P to the equation for n_k^C and the term γn_k^C to the equation for n_l^P, and changes nothing else; both added coefficients are nonnegative, so by (i) the new solution dominates the old one coordinatewise.

Finally, the price side of (4) is affine with constant coefficients, so the Jacobian $\partial(p^P, T)/\partial(n^P, n^C)$ is the constant matrix with -1 on the diagonal and $\alpha + \gamma$ off it, and the cross-effects vanish with $\alpha + \gamma$.

B Proofs

Proof of Proposition 1 (surplus and welfare)

Per region, provider surplus is the integral of net utility over participants. At interior solutions, the marginal provider is exactly indifferent, so $\tilde{v}^P = p^P - \alpha n^C - \gamma n^C_{lk} = 1 - n^P$ by (2), and

$$\int_{\tilde{v}^P}^{1} \big(v - \tilde{v}^P\big)\,\mathrm{d}v = \frac{(n^P)^2}{2}.$$

This holds regardless of α, γ and prices: they determine only where $\tilde{v}^P$, equivalently n^P, ends up, not the shape of the integral once n^P is fixed. However, it holds only when the participation constraint $n^P_k \leq 1$ is slack, otherwise there is no marginal provider.

This gives (9) region by region; summing over the two regions gives $W^P = (n^P)^2$, and the identical argument on the consumer side gives $W^C = (n^C)^2$. Welfare is the sum of the three parties' payoffs, and $\Pi_1 + \Pi_2 = 2\Pi(n^P, n^C)$ by symmetry giving $W = 2\Pi + W^P + W^C$; substituting (6) gives (8). Along the diagonal, $\mathrm{d}W/\mathrm{d}n = 4 - 4\big(1 - 2(\alpha+\gamma)\big)n > 0$ for $n \leq 1$ under Assumption 1. Likewise $\Pi(n,n) = 2n - 2\Delta n^2$ gives $\mathrm{d}\Pi/\mathrm{d}n = 2 - 4\Delta n$, positive on $\big[0, \frac{1}{2\Delta}\big)$ and negative beyond, which is the remaining monotonicity claim of the proposition.

Proof of Proposition 2 (destination principle)

Platform k sets $\big(p^P_k, p^C_k, \tau^C_{lk}\big)$ and collects $\Pi_k = p^P_k n^P_k + p^C_k n^C_k + \tau^C_{lk}\, n^C_l$, the last term only if region-l consumers cross, which by (1) they do if and only if

$$\tau^C_{lk} \;\leq\; \gamma\, n^P_k. \tag{26}$$

The fee τ^C_{kl} borne by platform k's own consumers is set by platform l and is taken as given by platform k.

Step 1. Each platform makes the other region cross. Without loss of generality, we focus on platform k's decision. Fix platform l's base fees p^C_l and p^P_l, and the additional fee τ^C_{kl} to be paid by consumers from region k to reach region l. By contradiction, consider an equilibrium in which platform k sets a crossing fee so high that consumers from region l do not cross. Then $U^P_k = v^P - p^P_k + \alpha n^C_k$, platform k collects no crossing revenue and its profit depends on the participation of the consumers and providers from its region. We will find a profitable deviation. If instead the platform had set $\tau^C_{lk} = 0$ and the same base fees as before, all consumers from region l would cross but platform k would not collect crossing revenues either, and we would have $U^P_k = v^P - p^P_k + \alpha n^C_k + \gamma n^C_{lk}$. The participation vector changes from one case to another. The fact that consumers from region l join increases the utility of providers from region k, which in turn increases the utility of consumers from region k. By fact (ii) of Appendix A, the participation vector increases component-wise, and so does the profit of platform k at unchanged fees. Thus, platform k never sets a crossing fee such that consumers from region l do not cross.

Step 2. The problem is concave and the constraint affine. Platform k's three instruments enter the participation system (23) through the columns of T_k, T_l and p^P_k, and the three participation levels it earns on are those of the corresponding rows, n^C_k, n^C_l and n^P_k. Writing $M_{[1:3]}$ for the 3×3 principal submatrix of M^{-1}, we have $\Pi_k = x^\top q$ with x the vector of instruments and $q = \text{const} - M_{[1:3]}x$, so the Hessian of Π_k is $-2M_{[1:3]}$. By Appendix A the matrix M is symmetric positive definite, hence so is M^{-1} and so is each of its principal submatrices; Π_k is therefore strictly concave. The constraint (26) which ensures multihoming is affine in x, since n_k^P is. Therefore, Karush–Kuhn–Tucker conditions are necessary and sufficient, and a candidate satisfying them with a nonnegative multiplier is platform k's global best response. Steps 3–6 construct the candidate on the hypothesis that (26) binds; Step 7 verifies the multiplier.

Step 3. The problem with the cross-border fee imposed. Substitute $\tau_{lk}^C = \gamma n_k^P$, which is platform k's own cap, the rival's τ_{kl}^C remaining a parameter. As in Step 1 the cross-border term in region l's consumer equation cancels, and the system becomes

$$\begin{aligned} n_k^P &= 1 - p_k^P + \alpha\, n_k^C + \gamma\, n_l^C, & n_l^P &= 1 - p_l^P + \alpha\, n_l^C + \gamma\, n_k^C, \\ n_k^C &= 1 - p_k^C - \tau_{kl}^C + \alpha\, n_k^P + \gamma\, n_l^P, & n_l^C &= 1 - p_l^C + \alpha\, n_l^P, \end{aligned} \tag{27}$$

with objective $\Pi_k = p_k^P n_k^P + p_k^C n_k^C + \gamma\, n_k^P\, n_l^C$. The two right-hand equations of (27) determine $\big(n_l^P, n_l^C\big)$ from n_k^C alone, and give

$$\frac{\mathrm{d}n_l^P}{\mathrm{d}n_k^C} = \rho = \frac{\gamma}{1-\alpha^2}, \qquad \frac{\mathrm{d}n_l^C}{\mathrm{d}n_k^C} = \alpha\rho, \qquad \frac{\mathrm{d}n_l^P}{\mathrm{d}n_k^P} = \frac{\mathrm{d}n_l^C}{\mathrm{d}n_k^P} = 0. \tag{28}$$

The two left-hand equations then invert, so platform k may choose $\big(n_k^P, n_k^C\big)$ and read its fees off

$$p_k^P = 1 - n_k^P + \alpha\, n_k^C + \gamma\, n_l^C, \qquad p_k^C = 1 - n_k^C - \tau_{kl}^C + \alpha\, n_k^P + \gamma\, n_l^P. \tag{29}$$

Step 4. First-order conditions. Write $\kappa := \gamma\rho$. Since n_l^P and n_l^C do not depend on n_k^P by (28),

$$\frac{\partial \Pi_k}{\partial n_k^P} = \big(1 - 2n_k^P + \alpha n_k^C + \gamma n_l^C\big) + \alpha n_k^C + \gamma n_l^C = 1 - 2n_k^P + 2\alpha\, n_k^C + 2\gamma\, n_l^C = 0,$$

that is $n_k^P = \frac{1}{2} + \alpha n_k^C + \gamma n_l^C$. Substituting into the first equation of (29),

$$p_k^P = 1 - \left[\tfrac{1}{2} + \alpha n_k^C + \gamma n_l^C\right] + \alpha n_k^C + \gamma n_l^C = \tfrac{1}{2}. \tag{30}$$

Differentiating in n_k^C and using (28),

$$\alpha\big(1+\kappa\big) n_k^P + p_k^C + \big(\kappa - 1\big) n_k^C + \alpha\kappa\, n_k^P = 0,$$

so $p_k^C = (1-\kappa) n_k^C - \alpha(1+2\kappa) n_k^P$; equating with the second equation of (29),

$$\big(2-\kappa\big)\, n_k^C = 1 + \big(\gamma\, n_l^P - \tau_{kl}^C\big) + 2\alpha\big(1+\kappa\big)\, n_k^P. \tag{31}$$

Step 5. Symmetry. In equilibrium platform l's cap binds as well, $\tau_{kl}^C = \gamma n_l^P$, so the bracket in (31) vanishes and the equilibrium conditions are, for $k = 1, 2$,

$$2n_k^P = 1 + 2\alpha\, n_k^C + 2\gamma\, n_l^C, \qquad (2-\kappa) n_k^C = 1 + 2\alpha\big(1+\kappa\big) n_k^P. \tag{32}$$

Let $\delta^P := n_1^P - n_2^P$ and $\delta^C := n_1^C - n_2^C$. Differencing the first equation of (32) across k gives $\delta^P = (\alpha - \gamma)\,\delta^C$, and differencing the second gives $(2-\kappa)\delta^C = 2\alpha(1+\kappa)\delta^P$, whence

$$\Big[2 - \kappa - 2\alpha\big(1+\kappa\big)\big(\alpha - \gamma\big)\Big]\, \delta^C = 0.$$

Under Assumption 1, $\kappa \le \alpha^2/(1-\alpha^2) \le \frac{1}{24}$ and $2\alpha(1+\kappa)(\alpha-\gamma) \le 2 \cdot \frac{1}{5} \cdot \frac{25}{24} \cdot \frac{1}{5} = \frac{1}{30}$, so the bracket exceeds $2 - \frac{1}{24} - \frac{1}{30} > \frac{19}{10}$ and $\delta^C = \delta^P = 0$.

Step 6. Closed form. With $n_1 = n_2$, (32) reads

$$n^{P,\mathrm{Dest}} = \tfrac{1}{2} + (\alpha+\gamma)\, n^{C,\mathrm{Dest}}, \qquad \bigl(2-\kappa\bigr)n^{C,\mathrm{Dest}} = 1 + 2\alpha\bigl(1+\kappa\bigr)n^{P,\mathrm{Dest}},$$

and eliminating $n^{P,\mathrm{Dest}}$,

$$n^{C,\mathrm{Dest}} = \frac{1+\alpha\bigl(1+\kappa\bigr)}{2-\kappa-2\alpha(\alpha+\gamma)\bigl(1+\kappa\bigr)}.$$

Multiplying numerator and denominator by $1-\alpha^2$ gives the form stated in the proposition. The denominator exceeds $2-\frac{1}{24}-2\cdot\frac{1}{5}\cdot\frac{2}{5}\cdot\frac{25}{24} > \frac{179}{100}$ under Assumption 1, so $n^{C,\mathrm{Dest}} > 0$. The fees are $p^{P,\mathrm{Dest}} = \frac{1}{2}$ by (30), $\tau^{\mathrm{Dest}} = \gamma\, n^{P,\mathrm{Dest}}$ by the binding cap, and $p^{C,\mathrm{Dest}} = 1 - n^{C,\mathrm{Dest}} + \alpha\, n^{P,\mathrm{Dest}}$ from the second equation of (29) with $\tau^C_{kl} = \gamma n^P_l$.

Step 7. The constraint is binding. Evaluating $\partial\Pi_k/\partial\tau^C_{lk}$ in the full system (23), at the fees of Step 6,

$$\frac{\partial\Pi_k}{\partial\tau^C_{lk}} = \frac{\bigl(1-\alpha^2+\gamma^2-3\gamma\bigr)\bigl[\bigl(1-\alpha^2\bigr)^2-\gamma^2\bigl(1+\alpha^2\bigr)\bigr]}{\bigl(1-(\alpha-\gamma)^2\bigr)\,\Delta\,\Omega^*} \text{ with } \Omega^* := \bigl(1-\alpha^2\bigr)\Bigl[2-\kappa-2\alpha(\alpha+\gamma)\bigl(1+\kappa\bigr)\Bigr].$$

Every factor of the denominator is positive. In the numerator, $\bigl(1-\alpha^2\bigr)^2-\gamma^2\bigl(1+\alpha^2\bigr) \geq \bigl(\frac{24}{25}\bigr)^2 - \frac{1}{25}\cdot\frac{26}{25} > 0$, while $1-\alpha^2+\gamma^2-3\gamma > 0$ if and only if $\alpha^2+3\gamma-\gamma^2 < 1$, which holds under Assumption 1 because $\alpha^2+3\gamma-\gamma^2 < \frac{1}{25}+\frac{3}{5} = \frac{16}{25}$. The derivative is therefore strictly positive. Since $\partial\bigl(\tau^C_{lk}-\gamma n^P_k\bigr)/\partial\tau^C_{lk} > 0$, the Karush–Kuhn–Tucker multiplier on (26) is that derivative divided by a positive number, hence strictly positive: the constraint is binding, and by Step 2 the candidate is platform k's global best response.

Step 8. Interiority and the two inequalities. From $n^{P,\mathrm{Dest}} = \frac{1}{2}+(\alpha+\gamma)n^{C,\mathrm{Dest}}$,

$$n^{P,\mathrm{Dest}} - n^{C,\mathrm{Dest}} = \tfrac{1}{2} - \Delta\, n^{C,\mathrm{Dest}} = \frac{\gamma\bigl(2-\gamma-2\alpha^2-2\alpha\gamma\bigr)}{2\,\Omega^*},$$

whose numerator is at least $\gamma\bigl(2-\frac{1}{5}-\frac{2}{25}-\frac{2}{25}\bigr) > 0$, so $n^{P,\mathrm{Dest}} > n^{C,\mathrm{Dest}}$.

Next, by $\tau^{\mathrm{Dest}} = \gamma n^{P,\mathrm{Dest}}$ and $p^{C,\mathrm{Dest}} = 1 - n^{C,\mathrm{Dest}} + \alpha n^{P,\mathrm{Dest}}$,

$$\tau^{\mathrm{Dest}} - p^{C,\mathrm{Dest}} = -\bigl(1-n^{C,\mathrm{Dest}}\bigr) - (\alpha-\gamma)\, n^{P,\mathrm{Dest}} \;<\; 0,$$

because $n^{C,\mathrm{Dest}} < 1$ and $\gamma < \alpha$.

Finally $n^{C,\mathrm{Dest}} \leq \bigl(1+\frac{1}{5}\cdot\frac{25}{24}\bigr)/\frac{179}{100} < \frac{17}{25}$ and $\alpha+\gamma < \frac{2}{5}$, so $n^{P,\mathrm{Dest}} < \frac{1}{2}+\frac{2}{5}\cdot\frac{17}{25} < 1$: participation is interior on both sides. □

Proof of Proposition 4 (monopoly)

Throughout, $x = (x_1, x_2) \in \{0,1\}^2$ denotes the crossing configuration, with $x_k = 1$ if the participating consumers of region k also join platform l, and $x_k = 0$ if none of them does. Write T_k for the total fee paid by a participating region-k consumer, so that $T_k = T^C_{kl} = p^C_k + \tau^C_{kl}$ when $x_k = 1$ and $T_k = p^C_k$ when $x_k = 0$. Participation in configuration x solves

$$n^P_k = 1 - p^P_k + \alpha\, n^C_k + \gamma\, x_l\, n^C_l, \qquad n^C_k = 1 - T_k + \alpha\, n^P_k + \gamma\, x_k\, n^P_l, \tag{33}$$

and the monopolist collects $\Pi^{\mathrm{Mono},x} = \sum_k \bigl(p^P_k n^P_k + T_k n^C_k\bigr)$.

Step 1. The four configurations, written in participation space. By Appendix A the system (33) is invertible for every x, so the monopolist may equivalently choose $n = (n_1^P, n_2^P, n_1^C, n_2^C)$ and recover the fees from

$$p_k^P = 1 - n_k^P + \alpha\, n_k^C + \gamma\, x_l\, n_l^C, \qquad T_k = 1 - n_k^C + \alpha\, n_k^P + \gamma\, x_k\, n_l^P. \tag{34}$$

Substituting (34) into the objective and collecting terms,

$$\Pi^{\mathrm{Mono},x}(n) = Q(n) + 2\gamma\big[\, x_2\, n_1^P n_2^C + x_1\, n_1^C n_2^P \,\big], \qquad Q(n) := \sum_k \Big[n_k^P + n_k^C - (n_k^P)^2 - (n_k^C)^2 + 2\alpha\, n_k^P n_k^C \Big]. \tag{35}$$

The four configurations therefore differ only in which of the two nonnegative terms outside of Q are present.

Step 2. Each configuration has a unique interior maximizer. The Hessian of (35) is $-2(I - \mathcal{B}_x)$, where $\mathcal{B}_x$ is the symmetric nonnegative matrix of externality coefficients of (33). Every row of $\mathcal{B}_x$ sums to at most $\alpha + \gamma$, so its spectral radius is at most $\alpha + \gamma < 1$ and the Hessian is negative definite. Hence $\Pi^{\mathrm{Mono},x}$ is strictly concave and has a unique maximizer n^x on $[0,1]^4$. If some coordinate of n^x were zero, say $n_1^P = 0$, then $\partial \Pi^{\mathrm{Mono},x} / \partial n_1^P = 1 + 2\alpha n_1^C + 2\gamma x_2 n_2^C \geq 1 > 0$ at that point, contradicting optimality; so all the components of n^x are positive.

Step 3. Configuration $x = (1,1)$ dominates. Let $x \neq (1,1)$. By (35),

$$\Pi^{\mathrm{Mono},(1,1)}(n^x) - \Pi^{\mathrm{Mono},x}(n^x) = 2\gamma\big[(1 - x_2)\, n_1^{P,x} n_2^{C,x} + (1 - x_1)\, n_1^{C,x} n_2^{P,x}\big] > 0,$$

since $\gamma > 0$, at least one of x_1, x_2 equals 0, and $n^x > 0$ by Step 2. Therefore

$$\max_n \Pi^{\mathrm{Mono},(1,1)}(n) \;\geq\; \Pi^{\mathrm{Mono},(1,1)}(n^x) \;>\; \Pi^{\mathrm{Mono},x}(n^x) = \max_n \Pi^{\mathrm{Mono},x}(n),$$

so no configuration other than $x = (1,1)$ can be optimal. Step 5 checks that the maximizer of $\Pi^{\mathrm{Mono},(1,1)}$ is supported by fees that do induce $x = (1,1)$.

Step 4. The maximizer at $x = (1,1)$. Here $\Pi^{\mathrm{Mono},(1,1)} = \Pi(n_1^P, n_1^C) + \Pi(n_2^P, n_2^C)$ with Π as in (6). The function is strictly concave by Step 2 and invariant both under exchanging the two regions and under exchanging the two sides. A strictly concave function has at most one maximizer; that maximizer is mapped to a maximizer by each of the two exchanges, hence is fixed by both, hence has $n_1^P = n_2^P = n_1^C = n_2^C =: n$. On that diagonal $\Pi^{\mathrm{Mono},(1,1)} = 2\Pi(n,n) = 4n - 4\Delta n^2$ by Proposition 1, and the first-order condition $4 - 8\Delta n = 0$ gives

$$n^{P,\mathrm{Mono}} = n^{C,\mathrm{Mono}} = \frac{1}{2\Delta}, \qquad \Pi^{\mathrm{Mono}} = \Pi\Big(\tfrac{1}{2\Delta}, \tfrac{1}{2\Delta}\Big) = \frac{1}{2\Delta} \quad \text{per platform, i.e. a joint profit of } \frac{1}{\Delta}.$$

Interiority: $1/2\Delta \leq 1$ is equivalent to $\alpha + \gamma \leq \frac{1}{2}$, and Assumption 1 gives $\alpha + \gamma < 2\alpha \leq \frac{2}{5}$. By (5), $p^{P,\mathrm{Mono}} = T^{\mathrm{Mono}} = 1 - \Delta/2\Delta = \frac{1}{2}$. Welfare follows from (7): $W^{\mathrm{Mono}} = 2\Pi(n,n) + 2n^2 = \frac{1}{\Delta} + \frac{1}{2\Delta^2} = \big(2\Delta + 1\big)/2\Delta^2$.

Step 5. The split, and consistency of the configuration. Only $T^{\mathrm{Mono}} = \frac{1}{2}$ is pinned down; any pair $(p^{C,\mathrm{Mono}}, \tau^{\mathrm{Mono}})$ with $p^{C,\mathrm{Mono}} + \tau^{\mathrm{Mono}} = \frac{1}{2}$ leaves (33) and the objective unchanged, which is Remark 3. Such a pair induces $x = (1,1)$ if and only if it satisfies $\tau^{\mathrm{Mono}} \geq 0$ and the crossing condition (1), that is

$$0 \;\leq\; \tau^{\mathrm{Mono}} \;\leq\; \gamma\, n^{P,\mathrm{Mono}} = \frac{\gamma}{2\Delta}.$$

This interval is nonempty, and contains $\tau^{\mathrm{Mono}} = 0$, so the maximum of Step 4 is attained. □

Proof of Proposition 5 (origin principle)

Platform k sets $\left(p_k^P, p_k^C, \tau_{kl}^C\right)$ and collects $p_k^P n_k^P + p_k^C n_k^C + \tau_{kl}^C n_k^C$ from its own two sides when its region crosses, and $p_k^P n_k^P + p_k^C n_k^C$ when it does not; in both cases its profit is $\Pi_k = p_k^P n_k^P + T_k n_k^C$, with T_k the total fee paid by a participating region-k consumer, as in the proof of Proposition 4.

Step 1. Platform k's problem in participation space. Fix the rival's fees $\left(p_l^P, p_l^C, \tau_{lk}^C\right)$ and suppose region l crosses. Write $a_l := 1 - p_l^P$ and $b_l := 1 - T_l$. The two region-l equations of (33) read

$$n_l^P = a_l + \alpha\, n_l^C + \gamma\, x_k\, n_k^C, \qquad n_l^C = b_l + \alpha\, n_l^P + \gamma\, n_k^P,$$

and, $\alpha < 1$, solve for $\left(n_l^P, n_l^C\right)$ as an affine function of $\left(n_k^P, n_k^C\right)$:

$$n_l^P = \frac{\left(a_l + \gamma x_k n_k^C\right) + \alpha\left(b_l + \gamma n_k^P\right)}{1-\alpha^2}, \qquad n_l^C = \frac{\alpha\left(a_l + \gamma x_k n_k^C\right) + \left(b_l + \gamma n_k^P\right)}{1-\alpha^2}. \tag{36}$$

At $x_k = 1$ this gives (11), with $\rho = \gamma/(1-\alpha^2)$. By Appendix A the remaining two equations invert, so platform k may choose $\left(n_k^P, n_k^C\right)$ and recover its fees from (34). Substituting,

$$\Pi_k(n_k^P, n_k^C) = n_k^P + n_k^C - (n_k^P)^2 - (n_k^C)^2 + 2\alpha\, n_k^P n_k^C + \gamma\, n_l^C\, n_k^P + \gamma\, x_k\, n_l^P\, n_k^C, \tag{37}$$

with n_l^P, n_l^C given by (36).

Step 2. Platform k makes its own region cross. Compare (37) at $x_k = 1$ and at $x_k = 0$, at the same $\left(n_k^P, n_k^C\right)$. By (36), $n_l^C\big|_{x_k=1} - n_l^C\big|_{x_k=0} = \alpha\gamma\, n_k^C/(1-\alpha^2)$, so

$$\Pi_k\big|_{x_k=1} - \Pi_k\big|_{x_k=0} = \frac{\alpha\gamma^2}{1-\alpha^2}\, n_k^P n_k^C + \gamma\, n_l^P\big|_{x_k=1}\, n_k^C.$$

Both terms are nonnegative. At $x_k = 0$ the objective is $n_k^P + n_k^C - (n_k^P)^2 - (n_k^C)^2 + 2\alpha n_k^P n_k^C + \gamma n_l^C n_k^P$ with n_l^C affine in n_k^P alone; its Hessian has diagonal $-2+2\kappa$ and -2 and off-diagonal 2α, negative definite because $1-\kappa > \alpha^2$ under Assumption 1, so the maximizer $\hat{n}_k$ is unique. Its coordinates are strictly positive: at $n_k^P = 0$ the partial derivative in n_k^P equals $1 + 2\alpha n_k^C + \gamma n_l^C \geq 1$, and at $n_k^C = 0$ the partial derivative in n_k^C equals $1 + 2\alpha n_k^P \geq 1$. Moreover $n_l^P \geq 1 - p_l^P > 0$ whenever the rival's provider fee is below one. Hence

$$\Pi_k\big|_{x_k=1}(\hat{n}_k) \;>\; \Pi_k\big|_{x_k=0}(\hat{n}_k) \;=\; \max\; \Pi_k\big|_{x_k=0},$$

so platform k does not choose $x_k = 0$. Setting $\tau_{kl}^C = 0$ makes (1) hold for any $n_l^P \geq 0$, so $x_k = 1$ is available to platform k whatever the rival does.

Step 3. Second-order condition. With $x_k = 1$, write $\kappa := \gamma\rho = \gamma^2/(1-\alpha^2)$. Differentiating (37) twice through (36),

$$\frac{\partial^2 \Pi_k}{\partial (n_k^P)^2} = \frac{\partial^2 \Pi_k}{\partial (n_k^C)^2} = -2 + 2\kappa, \qquad \frac{\partial^2 \Pi_k}{\partial n_k^P\, \partial n_k^C} = 2\alpha(1+\kappa).$$

The Hessian is negative definite if and only if $\kappa < 1$ and $1 - \kappa > \alpha(1+\kappa)$; the second condition rearranges to $(1-\alpha)^2 > \gamma^2$, that is $\alpha + \gamma < 1$, which holds under Assumption 1 and implies the first. Platform k's problem is therefore strictly concave and its first-order conditions are necessary and sufficient.

Step 4. First-order conditions. Differentiating (37) at $x_k = 1$, using (36),

$$1 - 2n_k^P + 2\alpha\, n_k^C + \gamma\, n_l^C + \kappa\, n_k^P + \alpha\kappa\, n_k^C = 0, \tag{38}$$
$$1 - 2n_k^C + 2\alpha\, n_k^P + \gamma\, n_l^P + \alpha\kappa\, n_k^P + \kappa\, n_k^C = 0, \tag{39}$$

for $k = 1, 2$: four linear equations in $\big(n_1^P, n_1^C, n_2^P, n_2^C\big)$.

Step 5. The solution is symmetric across regions and across sides. Set $\delta^P := n_1^P - n_2^P$ and $\delta^C := n_1^C - n_2^C$, and subtract (38) for $k = 2$ from (38) for $k = 1$, and likewise for (39). With $m := -2 + \kappa$ and $q := 2\alpha - \gamma + \alpha\kappa$ this gives

$$m\,\delta^P + q\,\delta^C = 0, \qquad q\,\delta^P + m\,\delta^C = 0,$$

which forces $\delta^P = \delta^C = 0$ unless $|q| = |m|$. Now $|m| = 2 - \kappa$, and

$$q < 2\alpha + \alpha\kappa < 2 - \kappa \iff \gamma^2 < (1 - \alpha)(2 - 2\alpha + \gamma),$$

which holds because $\gamma < 1-\alpha$; while if $q < 0$ then $|q| < \gamma < 2-\kappa$. Hence $|q| < |m|$ and $\delta^P = \delta^C = 0$. Imposing $n_1 = n_2$ and subtracting (39) from (38) gives

$$\big[-2 + \kappa - 2\alpha - \gamma - \alpha\kappa\big]\big(n^P - n^C\big) = 0,$$

and the bracket equals $-(2 + 2\alpha + \gamma) + \kappa(1 - \alpha) < 0$, so $n^P = n^C =: n$.

Step 6. Closed form. With $n_k^P = n_k^C = n_l^P = n_l^C = n$, (38) becomes $1 = n\big[2 - 2\alpha - \gamma - \kappa(1 + \alpha)\big]$ and $\kappa(1 + \alpha) = \gamma^2/(1 - \alpha)$, so

$$1 = n\Big[2 - 2\alpha - \gamma - \frac{\gamma^2}{1 - \alpha}\Big].$$

Multiplying by $1-\alpha$ and writing $u := 1-\alpha$, the bracket becomes $2u^2 - u\gamma - \gamma^2 = (2u+\gamma)(u-\gamma) = (2 - 2\alpha + \gamma)\Delta$, whence

$$n^{\text{Orig}} = \frac{1 - \alpha}{\Delta\,(2 - 2\alpha + \gamma)}, \qquad p^{P,\text{Orig}} = T^{\text{Orig}} = 1 - \Delta\, n^{\text{Orig}} = \frac{1 - \alpha + \gamma}{2 - 2\alpha + \gamma},$$

the price by (5).

Step 7. Interiority and the split. $n^{\text{Orig}} \le 1$ is equivalent to $u \le (2u + \gamma)(u - \gamma)$, that is to $u(2u - 1) \ge \gamma(u + \gamma)$. Under Assumption 1, $u \ge \frac{4}{5}$ gives $u(2u - 1) \ge \frac{12}{25}$, while $\gamma < \alpha \le \frac{1}{5}$ gives $\gamma(u + \gamma) < \frac{1}{5} \cdot \frac{6}{5} = \frac{6}{25}$; the inequality is strict. Finally $p^{P,\text{Orig}} \in (0, 1)$ and $n^{\text{Orig}} > 0$. As in Step 5 of the proof of Proposition 4, only T^{Orig} is determined, and a split $\big(p^{C,\text{Orig}}, \tau^{\text{Orig}}\big)$ induces crossing if and only if $0 \le \tau^{\text{Orig}} \le \gamma\, n^{\text{Orig}}$. □

Proof of Proposition 6 (origin versus destination)

Throughout, write $s := \alpha + \gamma$, $\Theta := 2 - 2\alpha + \gamma > 0$, $\Omega^* := \big(1 - \alpha^2\big)\big[2 - \kappa - 2\alpha(\alpha + \gamma)(1 + \kappa)\big] > 0$ as in the proof of Proposition 2, and $\bar{n} := \frac{1}{2}\big(n^P + n^C\big)$ for the average participation of a region-symmetric outcome. Let $w(n) := 4n - 2(1 - 2s)\, n^2$ denote welfare along the diagonal, which is strictly increasing on $[0, 1]$ under Assumption 1, by Proposition 1.

Step 1. Two decompositions. The quadratics (6) and (8) have Hessians $\begin{pmatrix} -2 & 2s \\ 2s & -2 \end{pmatrix}$ and $\begin{pmatrix} -2 & 4s \\ 4s & -2 \end{pmatrix}$, both diagonalized by the rotation $(n^P, n^C) \mapsto \big(\frac{n^P + n^C}{\sqrt{2}}, \frac{n^P - n^C}{\sqrt{2}}\big)$ with eigenvalues $-2(1 \mp s)$ and

$-2(1 \mp 2s)$. Expanding Π around its unconstrained maximizer $n^P = n^C = n^{\text{Mono}} = \frac{1}{2\Delta}$, and collecting the linear term of W as $4\bar{n}$, gives, for every region-symmetric outcome in which all consumers cross,

$$\Pi\big(n^P, n^C\big) = \Pi^{\text{Mono}} - 2\Delta\big(\bar{n} - n^{\text{Mono}}\big)^2 - \frac{1+s}{2}\,\big(n^P - n^C\big)^2, \tag{40}$$

$$W\big(n^P, n^C\big) = w(\bar{n}) - \frac{1+2s}{2}\,\big(n^P - n^C\big)^2. \tag{41}$$

Profit falls away from the monopoly point in both directions, and both profit and welfare are penalized by the spread $n^P - n^C$; away from the spread, welfare depends only on the average $\bar{n}$, monotonically.

Step 2. Three scalar facts. Substituting the closed forms of Propositions 2 and 5 and clearing the (positive) denominators yields the following identities, reproduced symbolically in `code/dataspaces/symbolic.py`:

$$\textbf{(F1)}\quad n^{\text{Orig}} - \bar{n}^{\text{Dest}} = \frac{\gamma^2\,\Xi_1}{4\,\Delta\,\Theta\,\Omega^*}, \qquad \textbf{(F2)}\quad n^{P,\text{Dest}} - n^{C,\text{Dest}} = \frac{\gamma\,\Xi_2}{2\,\Omega^*},$$

$$\textbf{(F3)}\quad n^{P,\text{Dest}} - n^{\text{Orig}} = \frac{\gamma\,\Xi_3}{2\,\Delta\,\Theta\,\Omega^*},$$

with numerators

$$\begin{aligned}
\Xi_1 &:= 4 + \gamma - 2\alpha - 4\alpha^2 - 5\alpha\gamma - \gamma^2 + 2\alpha^3 + 4\alpha^2\gamma + 2\alpha\gamma^2,\\
\Xi_2 &:= 2 - \gamma - 2\alpha^2 - 2\alpha\gamma,\\
\Xi_3 &:= 2 - 4\alpha - 4\gamma - \gamma^2 + 2\alpha\gamma + \gamma^3 + 3\alpha\gamma^2 + 6\alpha^2\gamma - 2\alpha^2\gamma^2 + 4\alpha^3 - 4\alpha^3\gamma - 2\alpha^4.
\end{aligned}$$

Bounding each numerator term by term with $0 < \gamma < \alpha \le \frac{1}{5}$,

$$\Xi_1 \;\ge\; 4-\tfrac{2}{5}-\tfrac{4}{25}-\tfrac{5}{25}-\tfrac{1}{25} \;>\; 3.2, \qquad \Xi_2 \;\ge\; 2-\tfrac{1}{5}-\tfrac{2}{25}-\tfrac{2}{25} \;>\; 1.6, \qquad \Xi_3 \;\ge\; 2-\tfrac{8}{5}-\tfrac{1}{25}-0.004 \;>\; 0.35,$$

so all three differences are strictly positive for $\gamma > 0$.

Step 3. Fees. $p^{P,\text{Dest}} = p^{P,\text{Mono}} = \frac{1}{2}$ by Propositions 2 and 4, and $p^{P,\text{Orig}} - \frac{1}{2} = T^{\text{Orig}} - \frac{1}{2} = \gamma/2\Theta > 0$ from Proposition 5. The comparison $T^{\text{Orig}} < T^{\text{Dest}}$ follows from the identity $T^{\text{Dest}} - T^{\text{Orig}} = \gamma\Psi/\big(2\,\Theta\,\Omega^*\big)$ with $\Psi := 2 + 4\gamma - 4\alpha^2 - 4\alpha\gamma - 4\alpha^2\gamma - 5\alpha\gamma^2 - \gamma^3 + 2\alpha^4 + 4\alpha^3\gamma + 2\alpha^2\gamma^2$, whose negative terms sum to less than 0.4 on the admissible box, so $\Psi > 1.6$.

Step 4. Participation. The chain $n^{C,\text{Dest}} < n^{\text{Orig}} < n^{P,\text{Dest}} < n^{\text{Mono}}$ follows from: (F2), which places $\bar{n}^{\text{Dest}}$ strictly between the two sides; (F1), which gives $\bar{n}^{\text{Dest}} < n^{\text{Orig}}$, hence $n^{C,\text{Dest}} < n^{\text{Orig}}$; (F3); and

$$n^{\text{Mono}} - n^{P,\text{Dest}} = \frac{\gamma\,(\alpha+\gamma)\,\Xi_2}{2\,\Delta\,\Omega^*} \;>\; 0.$$

Step 5. Profit. Apply (40) at each point. At the origin outcome the spread is zero, so, using $n^{\text{Mono}} - n^{\text{Orig}} = \gamma/(2\Delta\Theta)$,

$$\Pi^{\text{Mono}} - \Pi^{\text{Orig}} = 2\Delta\big(n^{\text{Mono}} - n^{\text{Orig}}\big)^2 = \frac{\gamma^2}{2\Delta\Theta^2} \;>\; 0.$$

And since $\bar{n}^{\text{Dest}} < n^{\text{Orig}} < n^{\text{Mono}}$ by (F1),

$$\Pi^{\text{Orig}} - \Pi^{\text{Dest}} = 2\Delta\Big[\big(n^{\text{Mono}} - \bar{n}^{\text{Dest}}\big)^2 - \big(n^{\text{Mono}} - n^{\text{Orig}}\big)^2\Big] + \frac{1+s}{2}\big(n^{P,\text{Dest}} - n^{C,\text{Dest}}\big)^2 \;>\; 0,$$

the bracket being a difference of squares of two positive numbers, the larger first, and the last term positive by (F2).

Step 6. Welfare. Apply (41). The origin and monopoly outcomes have zero spread, so $W^{\mathrm{Mono}} - W^{\mathrm{Orig}} = w\big(n^{\mathrm{Mono}}\big) - w\big(n^{\mathrm{Orig}}\big) > 0$ by Step 4 and w increasing. Finally

$$W^{\mathrm{Orig}} - W^{\mathrm{Dest}} = \Big[w\big(n^{\mathrm{Orig}}\big) - w\big(\bar{n}^{\mathrm{Dest}}\big)\Big] + \frac{1+2s}{2}\big(n^{P,\mathrm{Dest}} - n^{C,\mathrm{Dest}}\big)^2,$$

and both terms are strictly positive, by (F1) with w increasing, and by (F2): the origin principle delivers both a higher average participation and an even one. At $\gamma = 0$ every displayed difference carries a factor γ and vanishes. □

Proof of Proposition 8 (customs union)

Step 1. Configuration. The data space imposes $\tau^C_{12} = \tau^C_{21} = 0$. The crossing condition (1) then reads $\gamma n^P_l \geq 0$, which holds for every $n^P_l \geq 0$; with the tie-breaking convention of Section 2.1, every participating consumer crosses and no other configuration is consistent with $\tau^C_{kl} = 0$. The participation equations are therefore (2)–(3), and since the imposed pair (p^P, p^C) is common to the two regions, the unique solution of that system (Appendix A) is symmetric across regions.

Step 2. The programme in participation space. By (5) the map $(p^P, p^C) \mapsto (n^P, n^C)$ is an affine bijection, so the data space may choose $(n^P, n^C) \in (0,1]^2$ subject to (12), and maximizes W of (8).

Step 3. The feasible set is convex, the objective strictly concave. The Hessians of Π and of W in (n^P, n^C) are

$$\begin{pmatrix} -2 & 2(\alpha+\gamma) \\ 2(\alpha+\gamma) & -2 \end{pmatrix}, \qquad \begin{pmatrix} -2 & 4(\alpha+\gamma) \\ 4(\alpha+\gamma) & -2 \end{pmatrix},$$

negative definite when $\alpha + \gamma < 1$ and when $\alpha + \gamma < \frac{1}{2}$ respectively. Assumption 1 gives $\alpha + \gamma < 2\alpha \leq \frac{2}{5}$, so both hold. Hence $\{\Pi \geq \Pi^{\mathrm{Dest}}\}$ is convex and W is strictly concave.

Step 4. The solution is symmetric across sides. Both Π and W are invariant under exchanging n^P and n^C, and so is the constraint set and the box $(0,1]^2$. A strictly concave function on a convex set has at most one maximizer; the exchange maps that maximizer to a maximizer, hence fixes it, hence $n^P = n^C =: n$.

Step 5. The one-dimensional problem. On the diagonal $\Pi(n,n) = 2n - 2\Delta n^2$ and $W(n,n) = 4n - 2\big(1 - 2(\alpha+\gamma)\big)n^2$ by Proposition 1; $W(n,n)$ is strictly increasing on $[0,1]$, since $\mathrm{d}W/\mathrm{d}n = 4 - 4\big(1-2(\alpha+\gamma)\big)n \geq 4 - 4\big(1 - 2(\alpha+\gamma)\big) > 0$. The problem is therefore to maximize n subject to $2n - 2\Delta n^2 \geq \Pi^{\mathrm{Dest}}$ and $n \leq 1$. The constraint is $\Delta n^2 - n + \Pi^{\mathrm{Dest}}/2 \leq 0$, that is

$$n \in \Big[\frac{1-\sqrt{S}}{2\Delta},\ \frac{1+\sqrt{S}}{2\Delta}\Big], \qquad S = 1 - 2\Delta\,\Pi^{\mathrm{Dest}},$$

so that $n^{\mathrm{Cust}} = \min\big\{1, (1+\sqrt{S})/2\Delta\big\}$, and $p^{P,\mathrm{Cust}} = p^{C,\mathrm{Cust}} = 1 - \Delta n^{\mathrm{Cust}}$ by (5).

Step 6. $S \in (0,1)$. By Step 4 of the proof of Proposition 4, Π of (6) is strictly concave with unique maximizer $n^P = n^C = 1/2\Delta$, at which its value is $1/2\Delta$. By Proposition 2, $n^{P,\mathrm{Dest}} \neq n^{C,\mathrm{Dest}}$ whenever $\gamma > 0$, so the destination point is not that maximizer and $\Pi^{\mathrm{Dest}} < 1/2\Delta$, i.e. $S > 0$. Also $\Pi^{\mathrm{Dest}} \geq n^{P,\mathrm{Dest}}\big(1 - n^{P,\mathrm{Dest}}\big) + n^{C,\mathrm{Dest}}\big(1 - n^{C,\mathrm{Dest}}\big) > 0$ because both participation levels lie in $(0,1)$, so $S < 1$.

Step 7. Interiority. $(1+\sqrt{S})/2\Delta \leq 1$ requires $\sqrt{S} \leq 2\Delta - 1$, hence $\Delta > \frac{1}{2}$, and is then equivalent to $S \leq (2\Delta - 1)^2$. Since

$$(2\Delta - 1)^2 - S = 2\Delta\big[\Pi^{\mathrm{Dest}} - 2(\alpha + \gamma)\big],$$

this holds if and only if $\Pi^{\mathrm{Dest}} \geq 2(\alpha+\gamma)$, in which case $n^{\mathrm{Cust}} = (1+\sqrt{S})/2\Delta$ and $p^{P,\mathrm{Cust}} = p^{C,\mathrm{Cust}} = \big(1-\sqrt{S}\big)/2$. □

Proof of Proposition 9 (welfare ranking)

The comparisons $W^{\mathrm{Dest}} < W^{\mathrm{Orig}} < W^{\mathrm{Mono}}$ are part of Proposition 6, proved above. For $W^{\mathrm{Mono}} < W^{\mathrm{Cust}}$, both outcomes lie on the diagonal, $n^{\mathrm{Mono}} < n^{\mathrm{Cust}}$ by Proposition 8, and welfare along the diagonal is strictly increasing on $[0,1]$ by Proposition 1. Finally $W^{\mathrm{Cust}} < W^{\mathrm{Best}}$ because $n^{\mathrm{Cust}} < 1$: full participation caps each platform's profit at $\Pi(1,1) = 2(\alpha+\gamma)$, which is strictly below the outside option Π^{Dest} under Assumption 1 (the interiority condition of Proposition 8, which holds with slack), so the corner violates the platforms' participation constraint.

Preliminaries for the asymmetric environments

The remaining proofs concern Section 4. Throughout them, $\Gamma := \gamma_1 + \gamma_2$. Three facts are shared by all of them and stated once.

(i) The participation stage. With indexed parameters, the participation system keeps the structure of Appendix A: each row of the externality matrix has nonnegative entries summing to at most the relevant $\alpha + \gamma$, which is below $\frac{2}{5}$ under Assumption 2. The contraction argument therefore applies unchanged: for every fee vector and every crossing configuration, the participation stage has a unique outcome, and every fee lowers every participation level. The same device delivers positivity of equilibrium participation below: each system of first-order conditions can be arranged as $n = c + Kn$ with c strictly positive and K nonnegative with row sums below one, so its unique solution, $n = \sum_{m\geq 0} K^m c \;\geq c$, is strictly positive.

(ii) Configurations. The arguments by which each fee-setting party induces crossing — Step 1 of the proof of Proposition 2, Step 3 of the proof of Proposition 4, Step 2 of the proof of Proposition 5 — use only the positivity of participation and of the cross-region parameters, and carry over verbatim with indexed parameters. Every proof below therefore works in the configuration in which the consumers of both regions cross.

(iii) Binding caps under the destination principle. The verification that the crossing cap is active parallels Step 7 of the proof of Proposition 2. With asymmetric parameters, the positivity of $\partial\Pi_k/\partial\tau^C_{lk}$ at the candidate is verified numerically over the admissible parameter box (`code/checks/`); the analytic computation mirroring Step 7 is left for a future draft.

Proof of Proposition 10 (monopoly across regions)

By Remark 3 the monopolist's payoff depends on the consumer-side instruments only through the totals T^C_{kl}, so it chooses the four participation levels and recovers the fees from (13). Substituting (13) into the joint profit, in the configuration where both regions cross,

$$\Pi^{\mathrm{Mono}}(n) = \sum_k \Big[n^P_k + n^C_k - (n^P_k)^2 - (n^C_k)^2 + 2\alpha\, n^P_k n^C_k\Big] + \Gamma\,\big(n^P_1 n^C_2 + n^C_1 n^P_2\big),$$

which depends on (γ_1,γ_2) only through Γ. The Hessian is $-2(I - \mathcal{B})$ with $\mathcal{B}$ symmetric, nonnegative, and with rows summing to $\alpha + \Gamma/2 \leq \frac{2}{5} < 1$, so Π^{Mono} is strictly concave and has a unique maximizer, interior by the argument of Step 2 of the proof of Proposition 4. The objective is invariant under exchanging the two regions and under exchanging the two sides; the unique maximizer is fixed by both exchanges, hence $n_1^P = n_2^P = n_1^C = n_2^C =: n$. Any first-order condition then reads $1 - 2n + 2\alpha n + \Gamma n = 0$, so

$$n = \frac{1}{2 - 2\alpha - \Gamma} \ \le\ \frac{1}{2 - \frac{4}{5}} = \frac{5}{6} \ <\ 1,$$

using $2\alpha + \Gamma \le \frac{4}{5}$ under Assumption 2. From (13), $p_k^P = 1 - (1-\alpha-\gamma_k)\, n = (1-\alpha-\gamma_l)/(2-2\alpha-\Gamma)$, since $(2-2\alpha-\Gamma) - (1-\alpha-\gamma_k) = 1-\alpha-\gamma_l$, and identically for T_{kl}^C. The consumer-side split is free subject to $0 \le \tau_{kl}^C \le \gamma_k n$, a nonempty interval, so the crossing configuration is consistent, as in Step 5 of the proof of Proposition 4. By Proposition 1 region k's participants obtain $\frac{1}{2}(n_k^P)^2 + \frac{1}{2}(n_k^C)^2 = n^2$, equal across regions, and platform k collects $\Pi_k = \big(p_k^P + T_{kl}^C\big)n = 2(1-\alpha-\gamma_l)\, n^2$, whence $\Pi_1 - \Pi_2 = 2(\gamma_1 - \gamma_2)\, n^2$. $\square$

Proof of Proposition 11 (origin principle across regions)

The closed forms are established first; the surplus claims then follow from the signed differences and the comparison with monopoly in Step 6, together with $W_k = n_k^2$ from Proposition 1.

Step 1. Instruments and configuration. Platform k sets $\big(p_k^P, p_k^C, \tau_{kl}^C\big)$ and collects from its own two sides only, so by Remark 3 its payoff depends on the consumer-side instruments through $T_k := T_{kl}^C$ alone; the split $\tau_{kl}^C = 0$ implements any preferred T_k while making the crossing of its own region unconditional, and by fact (ii) of the preliminaries both regions cross.

Step 2. Rival response. Fix the rival's fees $\big(p_l^P, T_l\big)$. Region l's equations are $n_l^P = 1 - p_l^P + \alpha\, n_l^C + \gamma_l\, n_k^C$ and $n_l^C = 1 - T_l + \alpha\, n_l^P + \gamma_l\, n_k^P$; solving this 2×2 block, and writing $\rho_l := \gamma_l/(1-\alpha^2)$,

$$\frac{\partial n_l^P}{\partial n_k^C} = \rho_l, \qquad \frac{\partial n_l^P}{\partial n_k^P} = \alpha\rho_l, \qquad \frac{\partial n_l^C}{\partial n_k^P} = \rho_l, \qquad \frac{\partial n_l^C}{\partial n_k^C} = \alpha\rho_l.$$

Step 3. First-order conditions. Substituting (13), platform k's profit in participation space is

$$\Pi_k = n_k^P + n_k^C - (n_k^P)^2 - (n_k^C)^2 + 2\alpha\, n_k^P n_k^C + \gamma_k \left(n_l^C\, n_k^P + n_l^P\, n_k^C\right).$$

Write $\kappa := \gamma_1\gamma_2/(1-\alpha^2)$ and note $\gamma_k\rho_l = \kappa$ for both k. Using Step 2, the Hessian of Π_k has diagonal $-2 + 2\kappa$ and off-diagonal $2\alpha(1+\kappa)$; it is negative definite because $\kappa \le \frac{1}{25}\cdot\frac{25}{24} = \frac{1}{24}$ and $\alpha(1+\kappa) \le \frac{1}{5}\cdot\frac{25}{24} = \frac{5}{24} < 1 - \kappa$. The first-order conditions, for $k = 1,2$, are

$$1 - \big(2-\kappa\big)\, n_k^P + \alpha\big(2+\kappa\big)\, n_k^C + \gamma_k\, n_l^C = 0, \qquad 1 - \big(2-\kappa\big)\, n_k^C + \alpha\big(2+\kappa\big)\, n_k^P + \gamma_k\, n_l^P = 0. \quad (42)$$

Step 4. Symmetry across sides within each region. Set $\varepsilon_k := n_k^P - n_k^C$. Subtracting the two conditions of (42) for region k,

$$\Big[2 - \kappa + \alpha\big(2+\kappa\big)\Big]\, \varepsilon_k = -\,\gamma_k\, \varepsilon_l, \qquad k = 1,2.$$

If some ε_k were nonzero, both would be, and multiplying the two relations would give $\big[2 - \kappa + \alpha(2+\kappa)\big]^2 = \gamma_1\gamma_2$, which is impossible: the left side exceeds $\big(2 - \frac{1}{24}\big)^2 > 3$ while $\gamma_1\gamma_2 \le \frac{1}{25}$. Hence $\varepsilon_1 = \varepsilon_2 = 0$ and $n_k^P = n_k^C =: n_k$.

Step 5. Closed form. With $n_k^P = n_k^C = n_k$, and writing $u := 1-\alpha$, the first condition of (42) reads $\big[2 - 2\alpha - \kappa(1+\alpha)\big] n_k = 1 + \gamma_k n_l$; since $\kappa(1+\alpha) = \gamma_1\gamma_2/(1-\alpha)$, multiplying by u gives

$$\big(2u^2 - \gamma_1\gamma_2\big)\, n_k = u + u\,\gamma_k\, n_l, \qquad k = 1,2.$$

Write $G := 2u^2-\gamma_1\gamma_2$. Substituting one equation into the other, $\big(G^2-u^2\gamma_1\gamma_2\big)n_k = u\big(G+u\gamma_k\big)$, and expanding both sides shows $G^2 - u^2\gamma_1\gamma_2 = 4u^4 - 5u^2\gamma_1\gamma_2 + \gamma_1^2\gamma_2^2 = \big(u^2-\gamma_1\gamma_2\big)\big(4u^2-\gamma_1\gamma_2\big)$, every factor being positive since $u^2 \geq \frac{16}{25} > \gamma_1\gamma_2$. This gives the closed form announced in Proposition 11,

$$n_k^{\text{Orig}} = \frac{u\left(2u^2+\gamma_k\,u-\gamma_1\gamma_2\right)}{\big(u^2-\gamma_1\gamma_2\big)\big(4u^2-\gamma_1\gamma_2\big)},$$

strictly positive. For the fee, side symmetry within region k turns (13) into $p_k^P = 1 - u\,n_k + \gamma_k n_l$, and eliminating $\gamma_k n_l$ with the display above,

$$p_k^{P,\text{Orig}} = \frac{\big(u^2-\gamma_1\gamma_2\big)\,n_k}{u} = \frac{2u^2+\gamma_k\,u-\gamma_1\gamma_2}{4u^2-\gamma_1\gamma_2},$$

and identically for T_{kl}^C. The signed differences of Proposition 11 follow by subtraction, all denominators being positive: if $\gamma_1 > \gamma_2$,

$$n_1^{\text{Orig}} - n_2^{\text{Orig}} = \frac{u^2\,(\gamma_1-\gamma_2)}{\big(u^2-\gamma_1\gamma_2\big)\big(4u^2-\gamma_1\gamma_2\big)} \;>\; 0 \qquad \text{and} \qquad p_1^{P,\text{Orig}} - p_2^{P,\text{Orig}} = \frac{u\,(\gamma_1-\gamma_2)}{4u^2-\gamma_1\gamma_2} \;>\; 0,$$

the first being of first order in $\gamma_1-\gamma_2$.

Step 6. Comparison with monopoly, and interiority. Fix k and let $l \neq k$. The claim $n_k^{\text{Orig}} < n^{\text{Mono}} = 1/(2u-\Gamma)$ is, after clearing the (positive) denominators, the positivity of

$$\big(u^2-\gamma_1\gamma_2\big)\big(4u^2-\gamma_1\gamma_2\big)-u\left(2u^2+u\gamma_k-\gamma_1\gamma_2\right)\big(2u-\Gamma\big) = 2u^3\gamma_l+u^2\gamma_k^2-\gamma_l\Big(2u^2\gamma_k+u\gamma_k^2+u\gamma_k\gamma_l-\gamma_k^2\gamma_l\Big).$$

Under Assumption 2, $u \geq \frac{4}{5}$ and $\gamma_k, \gamma_l \leq \frac{1}{5}$, so $2u^3\gamma_l \geq \frac{128}{125}\gamma_l$ while the subtracted bracket is at most $\big(\frac{2}{5}+\frac{1}{25}+\frac{1}{25}\big)\gamma_l = \frac{60}{125}\gamma_l$; adding $u^2\gamma_k^2 > 0$, the display is strictly positive. Hence $n_k^{\text{Orig}} < n^{\text{Mono}} \leq \frac{5}{6} < 1$, which also delivers interiority. By Proposition 1, region k's participant surplus is n_k^2, so the surplus comparisons follow from the participation comparisons. Finally, only T_k is determined; a split induces crossing if and only if $0 \leq \tau_{kl}^C \leq \gamma_k\,n_l$, a nonempty interval containing $\tau_{kl}^C = 0$. □

Proof of Proposition 12 (destination principle across regions)

Step 1. Reduced system. Platform k sets $\big(p_k^P, p_k^C, \tau_{lk}^C\big)$ and collects $\Pi_k = p_k^P n_k^P + p_k^C n_k^C + \tau_{lk}^C\,n_l^C$, subject to the cap $\tau_{lk}^C \leq \gamma_l\,n_k^P$, which binds (preliminaries, facts (ii)–(iii)); the concavity-and-affine-constraint argument of Step 2 of the proof of Proposition 2 also carries over. Substituting platform k's own cap cancels the cross-border term in region l's consumer equation exactly:

$$n_l^C = 1 - p_l^C + \alpha\,n_l^P, \qquad n_l^P = 1 - p_l^P + \alpha\,n_l^C + \gamma_l\,n_k^C.$$

At fixed $\big(p_l^P, p_l^C\big)$ this block gives, with $\rho_l := \gamma_l/(1-\alpha^2)$,

$$\frac{\partial n_l^P}{\partial n_k^C} = \rho_l, \qquad \frac{\partial n_l^C}{\partial n_k^C} = \alpha\rho_l, \qquad \frac{\partial n_l^P}{\partial n_k^P} = \frac{\partial n_l^C}{\partial n_k^P} = 0.$$

Platform k may therefore choose $\big(n_k^P, n_k^C\big)$, read its fees off

$$p_k^P = 1 - n_k^P + \alpha\,n_k^C + \gamma_k\,n_l^C, \qquad p_k^C = 1 - n_k^C - \tau_{kl}^C + \alpha\,n_k^P + \gamma_k\,n_l^P,$$

the rival's τ_{kl}^C remaining a parameter, and maximize $\Pi_k = p_k^P n_k^P + p_k^C n_k^C + \gamma_l\,n_k^P\,n_l^C$.

Step 2. First-order conditions. Since n_l^P and n_l^C do not depend on n_k^P,

$$\frac{\partial \Pi_k}{\partial n_k^P} = \big(1 - 2n_k^P + \alpha\, n_k^C + \gamma_k\, n_l^C\big) + \alpha\, n_k^C + \gamma_l\, n_l^C = 1 - 2n_k^P + 2\alpha\, n_k^C + \Gamma\, n_l^C = 0,$$

which is the provider-side condition of the proposition; substituting it back into the fee, $2p_k^P = 2 - 2n_k^P + 2\alpha n_k^C + 2\gamma_k n_l^C = 1 + (\gamma_k - \gamma_l)\, n_l^C$. Differentiating in n_k^C, with $\kappa := \gamma_1\gamma_2/(1-\alpha^2)$ (again $\gamma_k \rho_l = \kappa$ for both k),

$$1 - \big(2-\kappa\big)n_k^C + \alpha\Big[2 + \Gamma\rho_l\Big]n_k^P + \big(\gamma_k\, n_l^P - \tau_{kl}^C\big) = 0,$$

and in equilibrium the rival's cap binds as well, so the last bracket vanishes:

$$\big(2-\kappa\big)\, n_k^C = 1 + \alpha\Big[2 + \frac{\Gamma\,\gamma_l}{1-\alpha^2}\Big]\, n_k^P, \qquad k = 1,2. \tag{43}$$

For the second-order condition, the Hessian of Π_k has entries -2, $-2+2\kappa$ on the diagonal and $\alpha\big(2+\Gamma\rho_l\big)$ off it; since $\kappa \le \frac{1}{24}$ and $\alpha\big(2+\Gamma\rho_l\big) \le \frac{1}{5}\big(2+\frac{1}{12}\big) = \frac{5}{12}$, we have $2\,(2-2\kappa) \ge \frac{23}{6} > \big(\frac{5}{12}\big)^2$ and the problem is strictly concave.

Step 3. The participation gaps. Write $\delta^P := n_1^P - n_2^P$ and $\delta^C := n_1^C - n_2^C$ for the participation gaps of Proposition 12. Subtracting the provider-side conditions across regions gives $2\delta^P = (2\alpha - \Gamma)\,\delta^C$; note $2\alpha - \Gamma > 0$ because $\gamma_k < \alpha$ for both k. Subtracting (43) across regions, and using the identity

$$\gamma_2\, n_1^P - \gamma_1\, n_2^P = \frac{\Gamma}{2}\,\delta^P - \frac{\gamma_1-\gamma_2}{2}\,\big(n_1^P + n_2^P\big),$$

we get

$$\big(2-\kappa\big)\,\delta^C = 2\alpha\,\delta^P + \frac{\alpha\,\Gamma}{1-\alpha^2}\Big[\frac{\Gamma}{2}\,\delta^P - \frac{\gamma_1-\gamma_2}{2}\,\big(n_1^P + n_2^P\big)\Big].$$

Substituting $\delta^P = \frac{2\alpha-\Gamma}{2}\delta^C$ and collecting terms yields the linear relation announced in Proposition 12,

$$\Lambda\,\delta^C = -\,\frac{\alpha\,\Gamma\,(\gamma_1-\gamma_2)}{2\,(1-\alpha^2)}\,\big(n_1^P + n_2^P\big),$$

with

$$\Lambda = 2 - \kappa - \Big(2\alpha + \frac{\alpha\,\Gamma^2}{2(1-\alpha^2)}\Big)\frac{2\alpha-\Gamma}{2} \;\ge\; 2 - \frac{1}{24} - \Big(\frac{2}{5} + \frac{1}{60}\Big)\frac{1}{5} \;=\; 2 - \frac{1}{24} - \frac{1}{12} \;=\; \frac{15}{8},$$

using $\kappa \le \frac{1}{24}$, $\alpha\Gamma^2/\big(2(1-\alpha^2)\big) \le \frac{1}{60}$ and $(2\alpha-\Gamma)/2 < \alpha \le \frac{1}{5}$. Since $n_1^P + n_2^P > 0$ (Step 4), $\gamma_1 > \gamma_2$ forces $\delta^C < 0$, hence $\delta^P < 0$.

Step 4. Positivity and interiority. Arrange the four equilibrium conditions as $n = c + K\,n$: $n_k^P = \frac{1}{2} + \alpha\, n_k^C + \frac{\Gamma}{2}\, n_l^C$ and $n_k^C = \big[1 + \alpha\big(2+\Gamma\rho_l\big)n_k^P\big]/(2-\kappa)$. The intercepts are strictly positive and the coefficient rows sum to at most $\max\big\{\alpha + \frac{\Gamma}{2},\ \frac{5}{12}/\frac{47}{24}\big\} \le \frac{2}{5} < 1$, so the unique solution is strictly positive (preliminaries, fact (i)). For interiority, let N^P and N^C denote the larger of the two provider and consumer levels. Then $N^P \le \frac{1}{2} + \frac{2}{5}N^C$ and $N^C \le \frac{24}{47}\big(1 + \frac{5}{12}N^P\big) = \frac{24}{47} + \frac{10}{47}N^P$, whence $N^P \le \frac{331}{430} < 1$ and $N^C \le \frac{1363}{2021} < 1$. $\square$

Proof of Proposition 13 (symmetric customs union across regions)

At $\gamma_1 = \gamma_2$ the common fee pair $\left(\frac{1}{2}, \frac{1}{2}\right)$ implements the monopoly outcome, and $\Pi^{\text{Mono}} > \Pi^{\text{Dest}}$ by Proposition 6, so both acceptance constraints hold with the common slack $\Pi^{\text{Mono}} - \Pi^{\text{Dest}} > 0$. At fixed fees, participation is the unique fixed point of the contraction of Appendix A and varies continuously with (γ_1, γ_2); hence so do the two profits at $\left(\frac{1}{2}, \frac{1}{2}\right)$. The outside options Π_k^{Dest} are continuous in (γ_1, γ_2) as well: the destination equilibrium solves the linear system of the proof of Proposition 12, whose coefficients are continuous and whose determinant is bounded away from zero under Assumption 2 (Step 4 there). The minimum slack at $\left(\frac{1}{2}, \frac{1}{2}\right)$ is therefore a continuous function of (γ_1, γ_2) that is strictly positive on the diagonal, so it remains positive in a neighbourhood of it: the symmetric customs union stays feasible for small asymmetries. Infeasibility under stronger asymmetry is established by Example 1, whose verification closes this appendix. □

Proof of Proposition 14 (regional customs union)

(i) A common base-fee pair is a region-specific fee vector whose two pairs coincide, so the regional customs union's feasible set contains the symmetric one's; over a larger feasible set the maximized welfare is weakly larger. (ii) The bound of Example 1 is derived, in its verification at the end of this appendix, for an *arbitrary* base-fee vector with $\tau_1 = \tau_2 = 0$ — common or region-specific, positive or negative: $\Pi_2 \le \left(1 + \gamma_2\right)^2 / \left(2(1 - \alpha)\right)$. Whenever $\left(1 + \gamma_2\right)^2 < 2(1 - \alpha)\,\Pi_2^{\text{Dest}}$, every admissible fee vector therefore violates platform 2's acceptance constraint, and the regional customs union is infeasible. □

Proof of Proposition 15 (monopoly across sides)

By Remark 3 the monopolist chooses participation. Write $e^P := 1 - c$ and $A := \alpha^P + \alpha^C$. In the configuration where both regions cross, substituting the inversion $p_k^P = e^P - n_k^P + \alpha^P n_k^C + \gamma^P n_l^C$, $T_k = 1 - n_k^C + \alpha^C n_k^P + \gamma^C n_l^P$ into the joint profit,

$$\Pi^{\text{Mono}}(n) = \sum_k \Big[e^P n_k^P + n_k^C - (n_k^P)^2 - (n_k^C)^2 + A\, n_k^P n_k^C \Big] + \big(\gamma^P + \gamma^C\big)\big(n_1^P n_2^C + n_1^C n_2^P\big),$$

and, as in Step 3 of the proof of Proposition 4, the crossing configuration $(1,1)$ dominates the other three, the crossing gain now carrying the factor $\gamma^P + \gamma^C$ instead of 2γ. The Hessian is $-2(I - \mathcal{B})$ with $\mathcal{B}$ symmetric, nonnegative, and with rows summing to $\Sigma/2 \le \frac{2}{5} < 1$, so the maximizer is unique; the objective is invariant under exchanging the two regions, so the maximizer is region-symmetric, $n_k^P = n^P$ and $n_k^C = n^C$. The first-order conditions reduce to

$$2\, n^P = e^P + \Sigma\, n^C, \qquad 2\, n^C = 1 + \Sigma\, n^P,$$

whence $n^P = \left(2e^P + \Sigma\right)/\left(4 - \Sigma^2\right)$, $n^C = \left(2 + \Sigma e^P\right)/\left(4 - \Sigma^2\right)$, and $n^P - n^C = \left(e^P - 1\right)/(2 + \Sigma) = -c/(2 + \Sigma)$. At $c = 0$ both equal $1/(2 - \Sigma)$. Interiority: since $e^P \le 1$ and $\Sigma \le \frac{4}{5}$, both levels are at most $(2 + \Sigma)/(4 - \Sigma^2) = 1/(2 - \Sigma) \le \frac{5}{6} < 1$. Fees follow from the inversion, and the consumer-side split is free subject to $0 \le \tau \le \gamma^C n^P$. □

Proof of Proposition 17 (destination principle across sides)

The architecture is that of the proof of Proposition 12; we carry general intercepts $e^P := 1 - c$ and $e^C := 1$, the proposition being the case $c = 0$. Write $D := 1 - \alpha^P \alpha^C$, $\rho := \gamma^P / D$ and $\kappa := \gamma^C \rho = \gamma^P \gamma^C / D$.

Step 1. Reduced system. The cap of platform k on region-l crossers is $\tau^C_{lk} \le \gamma^C n^P_k$ and binds. Substituting it cancels the cross-border term in region l's consumer equation:

$$n^C_l = e^C - p^C_l + \alpha^C n^P_l, \qquad n^P_l = e^P - p^P_l + \alpha^P n^C_l + \gamma^P n^C_k,$$

so that, at fixed $\big(p^P_l, p^C_l\big)$,

$$\frac{\partial n^P_l}{\partial n^C_k} = \rho, \qquad \frac{\partial n^C_l}{\partial n^C_k} = \alpha^C \rho, \qquad \frac{\partial n^P_l}{\partial n^P_k} = \frac{\partial n^C_l}{\partial n^P_k} = 0.$$

Platform k chooses $\big(n^P_k, n^C_k\big)$, reads its fees off $p^P_k = e^P - n^P_k + \alpha^P n^C_k + \gamma^P n^C_l$ and $p^C_k = e^C - n^C_k - \tau^C_{kl} + \alpha^C n^P_k + \gamma^C n^P_l$, and maximizes $\Pi_k = p^P_k n^P_k + p^C_k n^C_k + \gamma^C n^P_k n^C_l$.

Step 2. First-order conditions. Since n^P_l and n^C_l do not depend on n^P_k,

$$\frac{\partial \Pi_k}{\partial n^P_k} = \big(e^P - 2n^P_k + \alpha^P n^C_k + \gamma^P n^C_l\big) + \alpha^C n^C_k + \gamma^C n^C_l = 0,$$

that is $2n^P_k = e^P + A\, n^C_k + \big(\gamma^P + \gamma^C\big) n^C_l$ with $A := \alpha^P + \alpha^C$; substituting back, $2p^P_k = e^P + \big(\alpha^P - \alpha^C\big) n^C_k + \big(\gamma^P - \gamma^C\big) n^C_l$, which is the fee display of the proposition (and shows that a provider cost is absorbed at one half, as under monopoly). Differentiating in n^C_k,

$$e^C - \big(2-\kappa\big) n^C_k + \Big[A + \big(\gamma^P + \gamma^C\big)\alpha^C \rho\Big] n^P_k + \big(\gamma^C n^P_l - \tau^C_{kl}\big) = 0,$$

and in equilibrium the rival's cap binds, so the last bracket vanishes: $\big(2-\kappa\big) n^C_k = e^C + B\, n^P_k$, where $B := \alpha^P + \alpha^C + (\gamma^P + \gamma^C)\, \alpha^C \gamma^P / D$ collects the coefficient of n^P_k. The Hessian has diagonal -2, $-2 + 2\kappa$ and off-diagonal B; strict concavity follows from $\kappa \le \frac{1}{24}$ and $B \le \frac{2}{5} + \frac{1}{60} = \frac{25}{60}$, since $2(2 - 2\kappa) \ge \frac{23}{6} > B^2$.

Step 3. Symmetry across regions. Differencing the provider conditions across k gives $2\delta^P = \big(A - \gamma^P - \gamma^C\big)\delta^C$, and the consumer conditions give $\big(2-\kappa\big)\delta^C = B\,\delta^P$, whence $\big[2\big(2-\kappa\big) - B\big(A - \gamma^P - \gamma^C\big)\big]\delta^C = 0$. The bracket is at least $2 \cdot \frac{47}{24} - \frac{25}{60} \cdot \frac{2}{5} > 3$, so $\delta^C = \delta^P = 0$: the equilibrium is symmetric across regions; imposing symmetry, the provider-side condition of the proposition and the consumer-side condition $\big(2-\kappa\big) n^C = e^C + B\, n^P$ follow, with $n^C_l = n^C$.

Step 4. The two sides. With $e^P = e^C = 1$, subtracting the consumer condition from the provider condition gives the combined relation behind the comparison stated in the proposition, $\big(2 + B\big) n^P = \big(2 - \kappa + \Sigma\big) n^C$. Moreover

$$\Sigma - \kappa - B = \big(\gamma^P + \gamma^C\big)\big(1 - \alpha^C \rho\big) - \frac{\gamma^P \gamma^C}{D} \;\ge\; \big(\gamma^P + \gamma^C\big)\Big(\frac{23}{24} - \frac{5}{24}\Big) = \frac{3}{4}\,\big(\gamma^P + \gamma^C\big) > 0,$$

using $\alpha^C \rho \le \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{25}{24} = \frac{1}{24}$ and $\gamma^P \gamma^C / D \le \big(\gamma^P + \gamma^C\big) \cdot \frac{1}{5} \cdot \frac{25}{24} = \big(\gamma^P + \gamma^C\big)\frac{5}{24}$. Hence $n^{P,\mathrm{Dest}} > n^{C,\mathrm{Dest}}$, and the surplus comparison follows from Proposition 1.

Step 5. Positivity and interiority. Solving the two conditions, $\big(2 - \kappa - \frac{\Sigma B}{2}\big) n^C = e^C + \frac{B}{2} e^P$, with denominator at least $2 - \frac{1}{24} - \frac{1}{2} \cdot \frac{4}{5} \cdot \frac{25}{60} = \frac{43}{24} > 0$; both n^C and $n^P = \big(e^P + \Sigma n^C\big)/2$ are strictly positive, with $n^C \le \frac{29}{24} \big/ \frac{43}{24} = \frac{29}{43} < 1$ and $n^P \le \frac{1}{2}\big(1 + \frac{4}{5} \cdot \frac{29}{43}\big) = \frac{331}{430} < 1$. $\square$

Proof of Proposition 16 (origin principle across sides)

Step 1. Instruments and configuration. As in the proof of Proposition 11: platform k's payoff depends on its consumer-side instruments through T_k alone, the cap is slack, and both regions cross.

Step 2. Rival response. At fixed $\big(p_l^P, T_l\big)$, region l's block is $n_l^P = 1 - p_l^P + \alpha^P n_l^C + \gamma^P n_k^C$ and $n_l^C = 1 - T_l + \alpha^C n_l^P + \gamma^C n_k^P$, whence, with $D := 1 - \alpha^P\alpha^C$,

$$\frac{\partial n_l^P}{\partial n_k^C} = \frac{\gamma^P}{D}, \qquad \frac{\partial n_l^C}{\partial n_k^P} = \frac{\alpha^P\gamma^C}{D}, \qquad \frac{\partial n_l^C}{\partial n_k^P} = \frac{\gamma^C}{D}, \qquad \frac{\partial n_l^P}{\partial n_k^C} = \frac{\alpha^C\gamma^P}{D}.$$

Step 3. First-order conditions. Platform k's profit in participation space is

$$\Pi_k = n_k^P + n_k^C - (n_k^P)^2 - (n_k^C)^2 + A\, n_k^P n_k^C + \gamma^P n_l^C\, n_k^P + \gamma^C n_l^P\, n_k^C,$$

and, using Step 2, its first-order conditions are

$$\big(2-\kappa\big)n_k^P = 1 + \Big[A + \frac{\alpha^P(\gamma^C)^2}{D}\Big]n_k^C + \gamma^P n_l^C, \qquad \big(2-\kappa\big)n_k^C = 1 + \Big[A + \frac{\alpha^C(\gamma^P)^2}{D}\Big]n_k^P + \gamma^C n_l^P, \quad (44)$$

with $\kappa := \gamma^P\gamma^C/D$. The Hessian has diagonal $-2 + 2\kappa$ and off-diagonal $A + \big[\alpha^P(\gamma^C)^2 + \alpha^C(\gamma^P)^2\big]/D \le \frac{2}{5} + \frac{1}{60} < 2 - 2\kappa$, so the problem is strictly concave.

Step 4. Symmetry across regions. Differencing (44) across k,

$$\big(2-\kappa\big)\delta^P = q^P\,\delta^C, \qquad \big(2-\kappa\big)\delta^C = q^C\,\delta^P, \qquad q^P := A - \gamma^P + \frac{\alpha^P(\gamma^C)^2}{D}, \quad q^C := A - \gamma^C + \frac{\alpha^C(\gamma^P)^2}{D}.$$

Since $|q^P|, |q^C| \le \frac{2}{5} + \frac{1}{60} = \frac{25}{60} < 2 - \kappa$, combining the two relations forces $\delta^P = \delta^C = 0$.

Step 5. Closed form and signs. With region symmetry, (44) becomes the pair

$$\big(2-\kappa\big)\, n^{P,\mathrm{Orig}} = 1 + B^P\, n^{C,\mathrm{Orig}}, \qquad \big(2-\kappa\big)\, n^{C,\mathrm{Orig}} = 1 + B^C\, n^{P,\mathrm{Orig}},$$

where $B^P := \alpha^P + \alpha^C + \gamma^P + \alpha^P\big(\gamma^C\big)^2/D$ and $B^C := \alpha^P + \alpha^C + \gamma^C + \alpha^C\big(\gamma^P\big)^2/D$ collect the coefficients. The determinant $\big(2-\kappa\big)^2 - B^PB^C$ is at least $\big(\frac{47}{24}\big)^2 - \big(\frac{37}{60}\big)^2 > 3$, using $B^P, B^C \le \frac{3}{5} + \frac{1}{60} = \frac{37}{60}$, so the solution is unique,

$$n^{P,\mathrm{Orig}} = \frac{2-\kappa+B^P}{\big(2-\kappa\big)^2 - B^PB^C}, \qquad n^{C,\mathrm{Orig}} = \frac{2-\kappa+B^C}{\big(2-\kappa\big)^2 - B^PB^C},$$

strictly positive and, since the numerators are at most $2+\frac{37}{60} < 3$, strictly below one. The difference between the two sides is

$$n^{P,\mathrm{Orig}} - n^{C,\mathrm{Orig}} = \frac{\Psi}{\big(2-\kappa\big)^2 - B^PB^C}, \qquad \Psi := B^P - B^C = \big(\gamma^P - \gamma^C\big) + \frac{\alpha^P\big(\gamma^C\big)^2 - \alpha^C\big(\gamma^P\big)^2}{D},$$

so the gap has the sign of Ψ. For the particular cases: (i) at $\gamma^P = \gamma^C = \gamma$, $\Psi = \gamma^2\big(\alpha^P - \alpha^C\big)/D$; (ii) at $\alpha^P = \alpha^C = \alpha$, $\Psi = \big(\gamma^P - \gamma^C\big)\big[1 - \alpha\big(\gamma^P + \gamma^C\big)/D\big]$, and the bracket is at least $1 - \frac{1}{5}\cdot\frac{2}{5}\cdot\frac{25}{24} = \frac{11}{12} > 0$; (iii) if $\alpha^C \ge \alpha^P$ and $\gamma^C > \gamma^P$, then $\alpha^P(\gamma^C)^2 - \alpha^C(\gamma^P)^2 \le \alpha^P\big[(\gamma^C)^2 - (\gamma^P)^2\big]$, so $\Psi \le \big(\gamma^P - \gamma^C\big)\big[1 - \alpha^P\big(\gamma^P + \gamma^C\big)/D\big] < 0$, and symmetrically when the provider side dominates. The surplus comparisons follow from $W^P = (n^P)^2$ and $W^C = (n^C)^2$. Fees are recovered from the inversion (14), and the consumer-side split is free subject to $0 \le \tau \le \gamma^C n^{P,\mathrm{Orig}}$. $\square$

Proof of Proposition 19 (provider-welfare-maximizing customs union)

Maximizing n^P subject to $\Pi \geq \Pi^{\text{Dest}}$ is maximizing n^P subject to $\Pi_{\max}(n^P) \geq \Pi^{\text{Dest}}$, since any other n^C only shrinks the feasible range of n^P; the largest such n^P is the larger root of $\Pi_{\max}(n^P) = \Pi^{\text{Dest}}$. It exceeds the total-welfare customs union's n^P because that solves a different problem on the same feasible set, and it exceeds $n^{P,\text{Mono}}$ because $n^{P,\text{Mono}}$ is the unconstrained maximizer of $\Pi_{\max}$, which lies strictly between the two roots of any lower level set, and $\Pi^{\text{Dest}} < \Pi^{\text{Mono}}$. $\square$

Proof of Proposition 18 (participation costs, subsidies, and the first best)

(i) With regions symmetric and all consumers crossing, the total welfare of an allocation (n^P, n^C) in which the participants with the highest stand-alone values participate is, per region and summing the two sides' value integrals, the interaction terms and the cost,

$$W(n^P, n^C) = 2\Big[n^P - \tfrac{(n^P)^2}{2} - c\,n^P + n^C - \tfrac{(n^C)^2}{2} + \Sigma\, n^P n^C\Big],$$

valid at corners as well as in the interior. It is strictly concave ($\Sigma < 2$), with $\partial W/\partial n^C = 2\big[1 - n^C + \Sigma n^P\big] > 0$ always, so $n^C = 1$; and $\partial W/\partial n^P = 2\big[1 - c - n^P + \Sigma n^C\big]$, which at $(1, 1)$ equals $2(\Sigma - c)$. Full participation is optimal iff $c \leq \Sigma$; otherwise the provider-side optimum is interior, $n^P = 1 - c + \Sigma$.

(ii) A fee pair implements an allocation iff the marginal types are indifferent (or every participant strictly prefers to join, at a corner). On the provider side the marginal type is $\tilde{v}^P = 1 - n^P$ and joins iff $\tilde{v}^P - c - p^P + s^P n^C \geq 0$. At $n^P = n^C = 1$ this reads $p^P \leq s^P - c$: nonnegative fees work iff $c \leq s^P$, and for $s^P < c \leq \Sigma$ implementation requires $p^P \leq s^P - c < 0$ (note $s^P - c \geq s^P - \Sigma = -s^C$). For $c > \Sigma$, indifference of the type $\tilde{v}^P = c - \Sigma$ at $n^C = 1$ gives $p^P = (c - \Sigma) - c + s^P = -s^C$ exactly. The consumer corner is sustained by any $T \in [0, s^C n^P]$, zero included.

(iii) By the verification of Example 2 below, $p^{P,\text{Mono}} < 0$ iff $(1-c)\big(2 - \Sigma s^C\big) < s^C - s^P$. If $s^C \leq s^P$ the right side is nonpositive and the left strictly positive for $c < 1$, so the fee is never negative; if $s^C > s^P$ the condition is $c > c^*$. For the ordering, $c^* > \Sigma$ is equivalent to $(1 - \Sigma)\big(2 - \Sigma s^C\big) > s^C - s^P$; under Assumption 2, $s^P, s^C \leq \frac{2}{5}$, so $2 - \Sigma s^C \geq 2 - \frac{4}{5} \cdot \frac{2}{5} = \frac{42}{25}$ and it suffices that $\frac{42}{25}\big(1 - s^P - s^C\big) \geq s^C - s^P$, i.e. $\frac{42}{25} \geq \frac{17}{25} s^P + \frac{67}{25} s^C$, whose right side is at most $\frac{17+67}{25} \cdot \frac{2}{5} = \frac{168}{125} < \frac{42}{25}$. Finally $s^P \leq \Sigma$ trivially. $\square$

Verification of Example 2 (negative provider fees)

Write $s^P := \alpha^P + \gamma^P$ and $s^C := \alpha^C + \gamma^C$ for the externality intensity on each side. From the proof of Proposition 15, $n^{P,\text{Mono}} = \big(2e^P + \Sigma\big)/\big(4 - \Sigma^2\big)$ and $n^{C,\text{Mono}} = \big(2 + \Sigma e^P\big)/\big(4 - \Sigma^2\big)$, with $e^P := 1 - c$. Substituting into the inversion (16),

$$\begin{aligned}\big(4 - \Sigma^2\big)\, p^{P,\text{Mono}} &= e^P\big(4 - \Sigma^2\big) - \big(2e^P + \Sigma\big) + s^P\big(2 + \Sigma e^P\big) \\ &= e^P\big(2 - \Sigma^2 + s^P\Sigma\big) - \big(\Sigma - 2s^P\big) = e^P\big(2 - \Sigma\, s^C\big) - \big(s^C - s^P\big),\end{aligned}$$

using $\Sigma^2 - s^P\Sigma = \Sigma\, s^C$ and $\Sigma - 2s^P = s^C - s^P$. Since $4 - \Sigma^2 > 0$, the fee is negative exactly when $(1 - c)\big(2 - \Sigma s^C\big) < s^C - s^P$, and at fixed externality parameters this is a threshold in c, namely $c > 1 - \big(s^C - s^P\big)/\big(2 - \Sigma s^C\big)$. At the parameters of the example, $s^P = \frac{3}{40}$, $s^C = \frac{7}{20}$, $\Sigma = \frac{17}{40}$, $e^P = \frac{1}{10}$: the threshold is $c^* = \frac{1261}{1481}$, and at $c = \frac{9}{10}$,

$$n^P = \frac{1000}{6111} \approx 0.164, \qquad n^C = \frac{3268}{6111} \approx 0.535,$$

$$p^P = -\frac{719}{30555} \approx -0.024, \qquad T = 1 - n^C + s^C n^P \approx 0.523.$$

Both participation levels are interior, profit per platform is $p^P n^P + T n^C \approx 0.276 > 0$, and any split with $0 \le \tau \le \gamma^C n^P \approx 0.025$ sustains the crossing configuration. □

Verification of Example 1 (infeasible customs union)

Fix any base-fee vector with $\tau_1 = \tau_2 = 0$, common or region-specific. Both regions then cross, and platform 2 collects from its own participants only, $\Pi_2 = p_2^P n_2^P + p_2^C n_2^C$. Whether participation is interior or at the unit mass, the participation equations give $p_2^P \le 1 - n_2^P + \alpha\, n_2^C + \gamma_2\, n_1^C$ and $p_2^C \le 1 - n_2^C + \alpha\, n_2^P + \gamma_2\, n_1^P$, with equality in the interior case. Using $n_1^P, n_1^C \le 1$ and $n_2^P, n_2^C \ge 0$,

$$\Pi_2 \;\le\; \Big[\big(1+\gamma_2\big) - n_2^P + \alpha\, n_2^C\Big] n_2^P + \Big[\big(1+\gamma_2\big) - n_2^C + \alpha\, n_2^P\Big] n_2^C.$$

The right-hand side is a strictly concave quadratic in $\big(n_2^P, n_2^C\big)$ — Hessian diagonal -2, off-diagonal 2α — invariant under exchanging its two arguments, so its maximum lies on the diagonal, where it equals $2(1+\gamma_2)n - 2(1-\alpha)n^2$, maximized at $n = (1+\gamma_2)/\big(2(1-\alpha)\big)$ with value

$$\Pi_2 \;\le\; \frac{\big(1+\gamma_2\big)^2}{2\,\big(1-\alpha\big)}.$$

At $\alpha = \frac{1}{5}$, $\gamma_2 = \frac{1}{20}$ this is $\frac{441}{640} = 0.68906$, while the linear system of Proposition 12, evaluated at $\gamma_1 = \frac{3}{20}$, gives $\Pi_2^{\text{Dest}} = 0.70949$ (five decimals; `code/checks/` reproduces the value). Hence $\Pi_2 < \Pi_2^{\text{Dest}}$ for every admissible fee vector with free interconnection, and platform 2 refuses. □